Directions: You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

1. A sandwich shop offers 5 choices of bread, 3 choices of seasonings, 6 choices of meat. Also, the shop offers 8 toppings: lettuce, spinach, tomato, cheese, avocado, onion, cucumber, and bell pepper. To order a sandwich, the customer selects one bread, one seasoning, and one meat, plus any subset of the toppings (including no toppings at all and all 8 toppings).
(a) How many different sandwiches can be ordered at the shop? If you ate one sandwich a day and never repeated orders, how long would it take you to try them all?
(b) The shop owner decides that having both lettuce and spinach at the same time makes the sandwich too green, and makes a rule that a sandwich cannot have both of these toppings. How many different sandwiches can be ordered now?
2. How many 5 -digit ATM pin numbers have no repeated digits within distance 2 of one another? For example, 56759 counts because the pairs of 5 's are at distance 3 from one another. Similarly, 56756 counts, but 56765 does not count because the 6 's are too close to each other.
3. Repeated Consecutive Digits.
(a) How many 4-digit ATM pins have repeated consecutive digits? For example, 4412 and 4665 count, but 1234, 4564, and 4545 do not.
(b) Out of all 4 -digit ATM pins that have repeated digits, what percentage have repeated consecutive digits?
4. How many was are there to arrange the letters of 'ECCENTRIC':
(a) with no additional restrictions?
(b) beginning and ending with a C?
(c) beginning or ending with a C (or both)? (Note: CECENTRIC is allowed.)
(d) with all three C's consecutive?
5. Lattice paths from $(0,0)$ to $(8,4)$.

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(a) How many lattice paths are there from $(0,0)$ to $(8,4)$ in which each step increases one of the coordinates by 1 ?
(b) Suppose there is a deadly dragon that lives at the center $(4,2)$. How many lattice paths from $(0,0)$ to $(8,4)$ avoid the dragon?

