Directions: Solve 5 of the following 6 problems. All written work must be your own, using only permitted sources. See the "General Guidelines and Advice" on the homework page for more details.

1. Use network flows to prove Menger's Theorem for edge-disjoint paths in graphs: $\kappa'(x,y) = \lambda'(x,y)$. (Recall that $\kappa'(x,y)$ is the minimum size of a set of edges S such that $G - S$ has no xy-path, and $\lambda'(x,y)$ is the maximum size of a set of edge-disjoint xy-paths.)

2. Determine the smallest m such that every 2-connected graph with n vertices has a 2-connected spanning subgraph with at most m edges.

3. Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.

4. Let G be a graph with no induced copy of P_4, let $k = \omega(G)$, and let σ be an ordering of $V(G)$. Prove that with respect to σ, the greedy algorithm produces a proper k-coloring of G. Hint: show that if a vertex u receives color j, then u completes a j-clique with vertices that precede u in σ.

5. Let G be a graph not containing a 4-cycle. Prove that $\chi(G) \leq \alpha'(G) + 2$.

6. Let t be a nonnegative integer. For each n with $n \geq 5t$, construct an n-vertex graph with chromatic number $n - 2t$ and clique number $n - 3t$.