

1. Apply the existence and uniqueness theorems to the following. What can you conclude without solving the differential equation?

(a)  $y' = t \ln y$  with  $y(1) = 1$

(b)  $y' = (\ln t)y$  with  $y(1) = 1$

(c)  $(t - 6)y' + y = \sqrt{t + 1}$  with  $y(0) = 0$

(d)  $y' = |y|$  with  $y(0) = 0$ .

2. Solve  $y' + ty = ty^4$  using the Bernoulli eqn. substitution  $v = y^{1-n}$ .
3. A large-capacity fish tank initially contains 100 gal of water and 20 lbs of salt, but the salt concentration is too high. Fresh water is pumped in at a rate of 1 gal/min and the tank is drained at a rate of 0.5 gal/min. Assume the salt is well-mixed in the tank.
- (a) Write an equation for  $V(t)$ , where  $V$  is the volume of water (in gal) in the tank at time  $t$  (in minutes).
- (b) Write a differential equation for  $Q(t)$ , where  $Q$  is the quantity of salt in the tank (in lbs) at time  $t$  (in minutes).

(c) Solve for  $Q(t)$ .

(d) How long will it take for the tank to reach a concentration of 0.1 lbs of salt per gallon?