1. [10.4.16] Let $f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < 2 \end{cases}$. Give a sine series for the extension, and sketch the extension to which the series converges.

2. [10.5.5] If possible, use the method of separation of variables to convert the given partial differential equation to a pair of ordinary differential equations.

   (a) $u_{xx} + (x + y)u_{yy} = 0$

   (b) $u_{xx} + u_{yy} + xu = 0$

3. [10.5.12] A rod of 40cm with thermal diffusivity satisfying $\alpha^2 = 1$ has its ends maintained at 0 degrees. Initially, we have $u(x, 0) = x$ for $0 < x < 40$. Determine $u(x, t)$.

4. [10.4.36] Let $f(x) = x$ for $0 \leq x \leq 2$.

   (a) Sketch an even extension of f with period 4.

   (b) Find a cosine series for f.

   (c) Use part (b) to show that $\frac{\pi^2}{8} = \sum_{n\geq 1} \frac{1}{(2n-1)^2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$.

5. [10.5.22; Challenging] The heat conduction equation in two space dimensions is $\alpha^2(u_{xx} + u_{yy}) = u_t$. Assuming that $u(x, y, t) = X(x)Y(y)T(t)$, find a system of ordinary differential equations that are satisfied by X, Y, and T.

6. [10.1.8] Either solve the following boundary problem or show that it has no solution: $y'' + 4y = \sin x$ with $y(0) = 0$ and $y(\pi) = 0$.