1. [1.3.9] Verify that $y = 3t + t^2$ is a solution to $ty' - y = t^2$.

Note: $y' = 3 + 2t$

Check: $ty' - y \stackrel{?}{=} t^2$

$$t(3+2t) - (3t+t^2) \stackrel{?}{=} t^2$$

$$3t + 2t^2 - 3t - t^2 \stackrel{?}{=} t^2$$

$$t^2 \stackrel{?}{=} t^2 \quad \checkmark$$

So y is a soln to $ty' - y = t^2$

2. Given that $yx^2 + e^{yx} = x + 1$, find $\frac{dy}{dx}$ in terms of y and x.

Implicit Differentiation

$$\frac{d}{dx}\left[yx^2 + e^{yx}\right] = \frac{d}{dx}\left[x+1\right]$$

$$\frac{d}{dx}\left[yx^2\right] + e^{yx}\frac{d}{dx}\left[yx\right] = 1 + 0$$

$$\left(\frac{dy}{dx} \cdot x^2 + y \cdot 2x\right) + e^{yx}\left(\frac{dy}{dx} \cdot x + y \cdot 1\right) = 1$$

$$\frac{dy}{dx}\left(x^2 + xe^{yx}\right) = 1 - 2xy - ye^{yx}$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 2xy - ye^{yx}}{x^2 + xe^{yx}}}$$

3. [1.1.23] Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object itself and the temperature of its surroundings (the ambient air temperature in most cases). Suppose that the ambient temperature is 70°F and that the rate constant is $0.05(\text{min})^{-1}$.

(a) Write a differential equation for the temperature M as a function of time t.

$$\frac{dM}{dt} = k(70-M)$$

$$\frac{dM}{dt} = (0.05)(70-M)$$
$$= \frac{1}{20}(70-M)$$

$$\frac{dM}{dt} = \frac{1}{20}(70-M).$$

(b) Solve the initial value problem $M(0) = 100°$.

$$\int \frac{1}{70-M} \cdot \frac{dM}{dt}\, dt = \int \frac{1}{20}\, dt$$

$$-\ln|70-M| = \frac{1}{20}t + c$$

$$\ln|70-M| = C - \frac{1}{20}t$$

$$70 - M = Ce^{-\frac{1}{20}t}$$

$$M = 70 - Ce^{-\frac{1}{20}t}$$

Init. Condition:

$$100 = 70 - C\cdot \cancel{e^{0}}^{\,1} \qquad C = -30$$

$$\boxed{M(t) = 70 + 30e^{-\frac{1}{20}t}}$$

(c) How long will it take for the object to cool to 71°?

$$71 = 70 + 30e^{-\frac{1}{20}t}$$

$$\frac{1}{30} = e^{-\frac{1}{20}t}$$

$$\ln\left(\frac{1}{30}\right) = -\frac{1}{20}t$$

$$t = -20\ln\left(\frac{1}{30}\right)$$
$$= -20\ln\left((30)^{-1}\right)$$

$$= 20\ln(30)$$

$$\approx \boxed{68.024 \text{ min}}$$