Name: _____

Directions: Show all work. No credit for answers without work.

1. [**4 parts, 8 points each**] Compute the following.

(a) $\mathcal{L}\left\{1 + t^3 + te^t\right\}$

(c) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2 + 4s}\right\}$

(b) $\mathcal{L}\left\{2\sinh(3t) - u_5(t) \cdot t\right\}$

(d) $\mathcal{L}^{-1}\left\{\dfrac{s}{(s+6)^7}\right\}$

2. An undamped spring/mass system satisfies the equation $y'' + y = 0$. Initially, the system starts at rest. At time $t = 0$, an external motor is switched on and imparts a constant force of 1 unit. At time $t = 3$, the motor is turned off.

   (a) [**4 points**] Complete the IVP $y'' + y = g(t)$, $y(0) = 0$, $y'(0) = 0$ by expressing $g(t)$ in terms of step functions.

   (b) [**16 points**] Solve the IVP.

   (c) [**5 bonus points**] For $t \geq 3$, determine the amplitude of the oscillation exactly.

3. [**14 points**] Compute the inverse of $\begin{bmatrix} -2 & 1 & 1 \\ 9 & -4 & 0 \\ -8 & 4 & 2 \end{bmatrix}$.

4. [**14 points**] Find the eigenvector/eigenvalue pairs for the matrix $\begin{bmatrix} 3 & 2 \\ -13 & 1 \end{bmatrix}$.

5. Differential Equation System.

   (a) [**16 points**] Find the general solution to $\mathbf{x}' = \begin{bmatrix} 11 & -6 \\ 4 & -3 \end{bmatrix} \mathbf{x}$.

   (b) [**4 points**] Draw a phase portrait for the system in (a).