Name: ____________________

Directions: Show all work. No credit for answers without work. When possible, final answers should involve real numbers only.

1. [**10 points**] Solve the IVP $2y'' - y' - 3y = 0$ with $y(0) = 1$ and $y'(0) = 1$.

2. Consider Euler's method to approximate the solution to $y' = y$ passing through $(0, 1)$.

   (a) [**10 points**] With step size h and starting with $(x_0, y_0) = (0, 1)$, use Euler's method to compute (x_1, y_1), (x_2, y_2), and (x_3, y_3). Hint: factor your answer for y_2 and y_3.

   (b) [**5 points**] Use part (a) to give formulas for x_n and y_n in terms of n and h.

3. [**15 points**] Find the general solution to $y'' - 10y' + 29y = 0$.

4. [**15 points**] Find the general solution to $y^{(5)} + 2y^{(4)} - 3y^{(3)} = 0$.

5. [**5 points**] Write a differential equation whose general solution is $y = c_1 + c_2e^{-2t} + c_3te^{-2t}$.

6. [**15 points**] Find the general solution to $y'' - 3y' + 2y = 4e^{-t} + t$.

7. [**15 points**] Find the general solution to $y'' - y = e^t$.

8. An object with mass m, where $m > 1$, is attached to a spring. The resulting position function u satisfies the equation $mu'' + 2u' + u = 0$.

   (a) [**5 points**] Determine the quasi period as a function of mass m.

   (b) [**5 points**] Determine the mass that gives the shortest possible quasi period.