Name: ____

Directions: Show all work. No credit for answers without work.

1. [**2 parts, 15 points each**] Find the general solution explicitly.

   (a) $\dfrac{dy}{dx} = 2xy$

   (b) $y' + \dfrac{1}{2t-1}y = 1$

2. **[2 parts, 15 points each]** Solve the following IVPs, explicitly if possible.

   (a) $(2x + y\sec^2 x) + (3 + \tan x)\frac{dy}{dx} = 0$ with $y(0) = 1$

   (b) $\dfrac{dy}{dt} = t(y + t^2)$ with $y(0) = 0$

3. **[2 parts, 5 points each]** A drink chilled to 45°F is taken out of the refrigerator at time $t = 0$ and placed in a warm room. Newton's law of cooling states that an object cools (or warms) at a rate proportional to the difference between the temperature of the object and the temperature of its ambient environment; the drink has proportionality constant $k = \frac{1}{2}$ 1/(hours). Let $Q(t)$ be the temperature of the object (in °F) at time t (hours).

   (a) Suppose that the room's temperature is a constant 72°F. Write a differential equation for Q. Do not solve.

   (b) Suppose instead that the air conditioner is turned off at time $t = 0$ and the room temperature steadily rises from 72°F to 80°F over the course of 5 hours. Write a differential equation for Q, valid for $0 < t < 5$. Do not solve.

4. **[10 points]** Find all real numbers α such that $y = \alpha t$ is a solution to $\dfrac{dy}{dt} = \dfrac{y+t}{y-t}$.

5. **[2 parts, 5 points each]** If possible, apply the Existence and Uniqueness Theorems to the following differential equations; state the strongest conclusion given by the theorems.

   (a) $\dfrac{dy}{dt} = \dfrac{\sqrt{2y-t}}{t+1}$ with $y(2) = 1$

   (b) $(t+2)y' = (t+2)\ln(5-t) - y$, with $y(-3) = 5$

6. **[10 points]** Identify the equilibrium solution(s) of $y' = -(y^2 + 2y - 15)$, and classify each as stable, semistable, or unstable. Sketch the solutions (with phase line).