

Name: _____

Directions: Show all work. No credit for answers without work.

1. [**3 parts, 1 point each**] A retirement account earns an annual interest rate of 4%, compounded continuously. The retiree has monthly expenses of \$2000, which are withdrawn continuously. Let $S(t)$ be the dollar value of the account at time t (years), with $S(0) = S_0$.

(a) Write a differential equation for $S(t)$. (Note that t is measured in years but the given expenses are monthly.)

(b) Determine the equilibrium solution(s) to the equation in part (a) and classify each solution as stable, semistable, or unstable.

(c) What does your answer to part (b) mean in the context of the retirement account?

2. [**2 parts, 2 points each**] Apply the Existence and Uniqueness Theorems to the following differential equations; state the strongest conclusion given by the theorems.

(a) $(\cos t)y' + (\sin t)y = \frac{1}{2t+1}$, with $y(0) = 0$.

(b) $(y + 1)y' = \ln(t + 1)$, with $y(0) = 1$

3. [3 points] Find the general solution to $y' - \frac{1}{t}y = (\sin t)y^2$.