Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[3 parts, 1 point each]** Classify each equation by giving the order and stating whether or not it is linear.

   (a) $t^7y'' - e^ty^5 = \sin(t)$

   Second order (y''), nonlinear: (y^5).

   (b) $\cos(t)yy' = t$

   First order (y'), nonlinear (yy').

   (c) $\sin(t)y^{(3)} - \cos(t)y' = ty$

   Third order ($y^{(3)}$), linear.

2. **[3 points]** Find the general solution to $y' + \cos(t)y = \cos(t)$.

$$\mu = e^{\int \cos t\, dt} = e^{\sin t}$$

$$e^{\sin t}y' + \cos t\, e^{\sin t}y = \cos t\, e^{\sin t}$$

$$\frac{d}{dt}\left[e^{\sin t}y\right] = \cos t\, e^{\sin t}$$

$$e^{\sin t}y = \int \cos t \cdot e^{\sin t}\, dt$$

$u = \sin t$
$du = \cos t\, dt$

$$e^{\sin t}y = \int e^u\, du$$

$$e^{\sin t}y = e^u + C$$

$$e^{\sin t}y = e^{\sin t} + C$$

$$\boxed{y = 1 + Ce^{-\sin t}}$$

3. **[4 points]** Find ~~the~~ [illegible] solution to the following IVP explicitly ~~and determine the interval of validity~~:
$y' = (e^{-x} + e^{x})/(1+y)$ with $y(0) = 1$.

Seperable.

$$\int (1+y)\,dy = \int e^{-x} + e^{x}\,dx$$

$$y + \frac{y^2}{2} = -e^{-x} + e^{x} + C$$

Impose $y(0)=1$:

$$1 + \frac{1}{2} = -e^{0} + e^{0} + C$$

$$C = \frac{3}{2}$$

$$\frac{y^2}{2} + y + e^{-x} - e^{x} - C = 0$$

$$\frac{y^2}{2} + y + e^{-x} - e^{x} - \frac{3}{2} = 0$$

$$y^2 + 2y + 2e^{-x} - 2e^{x} - 3 = 0$$

$$y = \frac{-2 \pm \sqrt{4 - 4(2e^{-x} - 2e^{x} - 3)}}{2}$$

$$y = -1 \pm \sqrt{1 - 2e^{-x} + 2e^{x} + 3}$$

$$y = -1 \pm \sqrt{4 + 2e^{x} - 2e^{-x}}$$

To get $y(0)=1$, choose positive branch:

$$y = -1 + \sqrt{4 + 2e^{x} - 2e^{-x}}$$

$$y = -1 + \sqrt{4\left(1 + \frac{e^{x} - e^{-x}}{2}\right)}$$

$$\boxed{y = -1 + 2\sqrt{1 + \sinh(x)}}$$