Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[1 point]** For $y = \ln(\sin(t))$, find $\frac{dy}{dt}$.

$$\frac{dy}{dt} = \frac{d}{dt}\left[\ln(\sin(t))\right] = \frac{1}{\sin(t)} \cdot \frac{d}{dt}\left[\sin(t)\right] = \frac{\cos(t)}{\sin(t)} = \boxed{\cot(t)}$$

2. **[1 point]** Given $w = 3xy + x^2 + \sin(xy)$, compute $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.

$$\frac{\partial w}{\partial x} = \boxed{3y + 2x + y\cos(xy)}$$

$$\frac{\partial w}{\partial y} = 3x + 0 + x\cos(xy) = \boxed{3x + x\cos(xy)}$$

3. **[2 points]** Given $e^{ty} + y = \overset{-t}{\cancel{0}}$, find $\frac{dy}{dt}$ in terms of y and t.

$$\frac{d}{dt}\left[e^{ty} + y\right] = \frac{d}{dt}[t]$$

$$e^{ty}\frac{d}{dt}[ty] + \frac{dy}{dt} = -1$$

$$e^{ty}[y + ty'] + y' = -1$$

$$y'(te^{ty} + 1) = -1 - ye^{ty}$$

$$\frac{dy}{dt} = y' = \boxed{\frac{-1 - ye^{ty}}{1 + te^{ty}}}$$

4. **[3 parts, 2 points each]** Solve the following integrals.

(a) $\int t\cos(t^2)\,dt$

$= \frac{1}{2}\int \cos(t^2)\cdot 2t\,dt$

$= \frac{1}{2}\int \cos(u)\,du$

$= \frac{1}{2}\sin(u) + C$

$= \boxed{\frac{1}{2}\sin(t^2) + C}$

(b) $\int \frac{1}{x^2-4x+3}\,dx$

$\frac{1}{x^2-4x+3} = \frac{1}{(x-3)(x-1)} = \frac{A}{x-3} + \frac{B}{x-1}$

So: $1 = A(x-1) + B(x-3)$

$x=1 \Rightarrow 1 = -2B,\ B = -\frac{1}{2}$

$x=3 \Rightarrow 1 = 2A,\ A = \frac{1}{2}$

$\int \frac{1}{x^2-4x+3} = \int \frac{1}{2}\cdot\frac{1}{x-3}\,dx - \int \frac{1}{2}\frac{1}{x-1}\,dx$

$= \frac{1}{2}\ln|x-3| - \frac{1}{2}\ln|x-1| + C$

$= \boxed{\frac{1}{2}\ln\left|\frac{x-3}{x-1}\right| + C}$

(c) $\int t\ln t\,dt$

$u = \ln t \qquad v = \frac{t^2}{2}$

$du = \frac{1}{t}\,dt \qquad dv = t\,dt$

$\int t\ln t\,dt = uv - \int v\,du$

$= \frac{t^2}{2}\ln t - \int \frac{t^2}{2}\cdot\frac{1}{t}\,dt$

$= \frac{t^2}{2}\ln t - \frac{1}{2}\int t\,dt$

$= \boxed{\frac{t^2}{2}\ln t - \frac{1}{4}t^2 + C}$