Directions: Solve 5 of the following 6 problems. All written work must be your own, using only permitted sources. See the "General Guidelines and Advice" on the homework page for more details.

1. Prove that the complement of a disconnected graph is connected.

2. Find a P_6-decomposition of the Petersen graph or show that no such decomposition exists.

3. In a graph, a uv-path is a path with endpoints u and v. Prove that if G has a uv-path of length at least k^2 and some cycle in G contains u and v, then G has a cycle of length at least $k + 1$.

4. Let G be a graph with girth 4 in which every vertex has degree k. Prove that G has at least $2k$ vertices. Determine all such graphs with exactly $2k$ vertices.

5. Prove that a self-complementary graph with n vertices exists if and only if $n = 4k$ or $n = 4k+1$ for some integer k. Hint: When n is divisible by 4, generalize the structure of P_4 by splitting the vertices into four groups. For n of the form $n = 4k + 1$, add one vertex to the graph constructed for $n = 4k$.

6. Let G be a simple graph in which every vertex has degree 3. Prove that G decomposes into claws if and only if G is bipartite.