Directions: Solve 5 of the following 6 problems. All written work must be your own, using only permitted sources. See the "General Guidelines and Advice" on the homework page for more details.

1. [IGT 1.1.10] Prove or disprove: the complement of a disconnected graph is connected.

2. Find a P_6-decomposition of the Petersen graph or show that no such decomposition exists.

3. [IGT 1.1.14] Prove that removing opposite corner squares from an 8-by-8 checkerboard leaves a subboard that cannot be partitioned into 1-by-2 and 2-by-1 rectangles. Hint: use a bipartite graph to model the problem.

4. [IGT 1.1.26] Let G be a graph with girth 4 in which every vertex has degree k. Prove that G has at least $2k$ vertices. Determine all such graphs with exactly $2k$ vertices.

5. [IGT 1.1.31] Prove that a self-complementary graph with n vertices exists if and only if $n = 4k$ or $n = 4k + 1$ for some integer k. Hint: When n is divisible by 4, generalize the structure of P_4 by splitting the vertices into four groups. For n of the form $n = 4k + 1$, add one vertex to the graph constructed for $n = 4k$.

6. [IGT 1.1.38] Let G be a simple graph in which every vertex has degree 3. Prove that G decomposes into claws if and only if G is bipartite.