Name: $\qquad$
Directions: Show all work. No credit for answers without work.

1. Poker hands.
(a) [ $\mathbf{2}$ points] How many poker hands have 3 clubs, 1 heart, and 1 diamond?
(b) [2 points] How many poker hands have 5 cards of distinct ranks? For example, 3H 4H 5 H 6 H 7 H counts but 3 H 3 S 4 D 5 C 6 C does not.
(c) [2 points] In a standard deck of cards, the hearts and diamonds are red and the clubs and spades are black. How many poker hands have at least one card in each color?
2. Lattice paths from $(0,0)$ to $(9,5)$. Recall that each step of a lattice path increases one of the coordinates by 1 ; geometrically, we either move one unit in the horizontal direction or 1 unit in the vertical direction.

(a) [2 points] How many lattice paths are there from $(0,0)$ to $(9,5)$ ?
(b) [1 point] How many lattice paths from $(0,0)$ to $(9,5)$ move in the horizontal direction for their last step? (One such path is highlighted in bold above.)
(c) [1 point] How many lattice paths from $(0,0)$ to $(9,5)$ never move in the vertical direction twice in a row? (One such path is highlighted in bold above.)
