

1. A population of ants grows logistically. Initially, the ant population is 10% of the carrying capacity. After 1 year, the ant population has doubled. Compute

- (a) The population (as a percentage of carrying capacity) after 2 years.
- (b) The time at which the population reaches 90% of carrying capacity.
- (c) The time at which the population is increasing fastest.

*Hint:* recall the logistic equation  $\frac{dy}{dt} = r(1 - (y/K))y$ . Let  $y(t)$  be the population of ants in units of carrying capacity, so that  $K = 1$ ,  $y(0) = 0.1$ , and  $y(1) = 0.2$ .

2. Find an integrating factor  $\mu(x)$  that depends only on  $x$  to solve

$$\frac{dy}{dx} = - \left( \frac{y \sin x + 2yx(\cos x)}{x \sin x} \right).$$

*Hint:* rewrite the equation in standard differential form. After transforming to an exact equation, try imposing  $\psi = N_y$  first.

3. Compute the following.

(a)  $\frac{3+2i}{4-i}$

(b)  $(2+i)e^{1-\frac{\pi}{2}i}$