- 1. Apply the existence and uniqueness theorems to the following. What can you conclude without solving the differential equation?
  - (a)  $y' = t \ln y$  with y(1) = 1

(b)  $y' = (\ln t)y$  with y(1) = 1

(c)  $(t-6)y' + y = \sqrt{t+1}$  with y(0) = 0

(d) y' = |y| with y(0) = 0.

2. Solve  $y' + ty = ty^4$  using the Bernoulli eqn. substitution  $v = y^{1-n}$ .

- 3. A large-capacity fish tank initially contains 100 gal of water and 20 lbs of salt, but the salt concentration is too high. Fresh water is pumped in at a rate of 1 gal/min and the tank is drained at a rate of 0.5 gal/min. Assume the salt is well-mixed in the tank.
  - (a) Write an equation for V(t), where V is the volume of water (in gal) in the tank at time t (in minutes).

(b) Write a differential equation for Q(t), where Q is the quantity of salt in the tank (in lbs) at time t (in minutes).

(c) Solve for Q(t).

(d) How long will it take for the tank to reach a concentration of 0.1 lbs of salt per gallon?