Solutions

1. Give the general solution to the differential equation $y' = \frac{x^2}{\ln y}$. Hint: if needed, integrate $\int \ln y\, dy$ by parts with $u = \ln y$ and $dv = dy$.

$$\int \ln y\, dy = \int x^2\, dx$$

$$y \ln y - y = \frac{x^3}{3} + C$$

$$\boxed{3y\ln y - 3y = x^3 + C}$$

2. [2.2.{15,20}] Solve the following IVPs explicitly.

   (a) $y' = 2x/(1 + 2y)$ with $y(2) = 0$

$$\int (1+2y)\, dy = \int 2x\, dx$$

$$y + y^2 = x^2 + C$$

$y(2)=0$:

$$0 + 0^2 = 2^2 + C$$

$$C = -4$$

$$y^2 + y + 4 - x^2 = 0$$

$$y = \frac{-1 \pm \sqrt{1 - 4(4-x^2)}}{2} = \frac{-1 \pm \sqrt{4x^2 - 15}}{2}$$

For $y(2)=0$, choose $\boxed{y = \frac{-1 + \sqrt{4x^2-15}}{2}}$.

   (b) $y^2(1 - x^2)^{1/2}dy = \arcsin x\, dx$ with $y(0)=1$

$$\int y^2\, dy = \int \frac{\arcsin x}{\sqrt{1-x^2}}\, dx$$

$$\frac{y^3}{3} = \int u\, du \qquad u = \arcsin x, \quad du = \frac{1}{\sqrt{1-x^2}}\, dx$$

$$\frac{y^3}{3} = \frac{u^2}{2} + C$$

$$\frac{y^3}{3} = \frac{(\arcsin x)^2}{2} + C$$

$y(0)=1$:

$$\frac{1}{3} = \frac{0^2}{2} + C, \quad C = \frac{1}{3}$$

$$\frac{y^3}{3} = \frac{(\arcsin x)^2}{2} + \frac{1}{3}$$

$$y = \left[1 + \frac{3}{2}(\arcsin x)^2\right]^{1/3}$$

3. [2.2.24] Solve the IVP $y' = (2 - e^x)/(3 + 2y)$ with $y(0) = 0$ and determine where the solution attains its maximum value.

$$\int (3 + 2y)\,dy = \int (2 - e^x)\,dx$$

$$3y + y^2 = 2x - e^x + C$$

$y(0)=0$:

$$0 + 0^2 = 0 - 1 + C; \quad C = 1$$

$$\boxed{3y + y^2 = 2x - e^x + 1}$$

· In interval of validity:

$3 + 2y$ is positive

So, the soln increases when

$2 - e^x > 0$

and decreases when $2 - e^x < 0$.

Max value is attained when

$2 - e^x = 0$

$$\boxed{x = \ln 2}$$

4. [2.2.21] Solve the IVP $y' = (1 + 3x^2)/(3y^2 - 6y)$ with $y(0) = 1$ and determine the interval in which the solution is valid, ~~approximately~~ .

$$\int (3y^2 - 6y)\,dy = \int (1 + 3x^2)\,dx$$

$$y^3 - 3y^2 = x + x^3 + C$$

$y(0)=1$:

$$1 - 3(1) = 0 + 0 + C$$

$$C = -2$$

$$\boxed{y^3 - 3y^2 = x + x^3 - 2}$$

· Valid when $3y^2 - 6y \neq 0$

$3y(y - 2) \neq 0$

$y \neq 0$ and $y \neq 2$.

· Valid when y in range $(0, 2)$.

· $y = 0 \Rightarrow x^3 + x - 2 = 0$

$x \approx 1$

· $y = 2 \Rightarrow x^3 + x - 2 = 8 - 3 \cdot 4$

$x^3 + x = -2$

$x = -1$

$$\boxed{\text{So the soln is valid for } x \text{ in the range } (-1, 1).}$$