WS 0 Solutions

1.

(a) $\boxed{\frac{du}{dt} = 2te^{3t} + 3t^2e^{3t}}$

(b) $\frac{dv}{dt} = e^{\tan t} \cdot \frac{d}{dt}[\tan t]$

$= \boxed{(\sec^2 t)\, e^{\tan t}}$

(c) $\frac{d}{dt}[\sinh t] = \frac{d}{dt}\left[\frac{1}{2}(e^t - e^{-t})\right]$

$= \frac{1}{2}(e^t + e^{-t}) = \boxed{\cosh t}$

(d) $\frac{d}{dt}[\cosh t] = \frac{d}{dt}\left[\frac{1}{2}(e^t + e^{-t})\right]$

$= \frac{1}{2}(e^t - e^{-t}) = \boxed{\sinh t}$

2. (a) $\int \frac{x}{x^2-4}\,dx$ $\quad u = x^2-4$, $\quad du = 2x\,dx$

$$= \frac{1}{2}\int \frac{1}{x^2-4}\cdot 2x\,dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|x^2-4| + C}$$

(b) $\int \frac{1}{x^2-4}\,dx = \int \frac{1}{(x+2)(x-2)}\,dx$

Use partial fractions:

$$\frac{1}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2},$$ where $A(x-2)+B(x+2)=1$

Subst $x=2$: $4B=1$, $B=\frac{1}{4}$

$x=-2$: $-4A=1$, $A=-\frac{1}{4}$

$$\int \frac{1}{(x+2)(x-2)}\,dx = \int -\frac{1}{4}\cdot\frac{1}{x+2} + \frac{1}{4}\cdot\frac{1}{x-2}\,dx$$

$$= -\frac{1}{4}\ln|x+2| + \frac{1}{4}\ln|x-2| + C$$

$$= \boxed{\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C}$$

(c) $\displaystyle\int \frac{1}{x^2+4}\,dx$ $\qquad \left.\begin{array}{l} x = 2\tan\theta \\ dx = 2\sec^2\theta \end{array}\right\}$ $\qquad \theta = \arctan\left(\frac{x}{2}\right)$

$$= \int \frac{1}{4\tan^2\theta + 4}\cdot 2\sec^2\theta\,d\theta$$

$$= \frac{1}{4}\int \frac{1}{\tan^2\theta+1}\,2\sec^2\theta\,d\theta$$

$$= \frac{1}{2}\int \frac{1}{\cancel{\sec^2\theta}}\cdot\cancel{\sec^2\theta}\,d\theta$$

$$= \frac{1}{2}\int d\theta = \frac{1}{2}\theta + C = \boxed{\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$$

arctan sometimes denoted by $\tan^{-1}$

3. $$\frac{\partial y}{\partial u} = 2u e^{v} + 3\cos(3u+2v)$$

$$\frac{\partial y}{\partial v} = u^{2} e^{v} + 2\cos(3u+2v)$$

4. $$\frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt}$$

$$= \boxed{(ye^{xy})\cdot(\ln t + 1) + (xe^{xy})(e^{t})}$$

5. $\frac{dV}{dt} = -k_0 S$, where S is the surface area, and k_0 is a positive constant.

Since $V = \frac{4}{3}\pi r^3$, we have that $r = \left(\frac{3}{4\pi}V\right)^{1/3}$.

Therefore $S = 4\pi r^2 = 4\pi\left(\frac{3}{4\pi}V\right)^{2/3}$.

So $$\frac{dV}{dt} = -k_0 S = -\boxed{k_0 \cdot 4\pi \cdot \left(\frac{3}{4\pi}\right)^{2/3}}\cdot V^{2/3} = -\boxed{k}\,V^{2/3}.$$

Therefore $\boxed{\frac{dV}{dt} = -kV^{2/3}}$, where k is a pos. const.