

Name: _____

Directions: Show all work. No credit for answers without work.

1. [24 points] A mass of 1 kg stretches a spring by 5 cm. The spring/mass system is enclosed in a medium which imparts a viscous force of magnitude 24 N when the mass moves at a velocity of 2 m/s. An external motor imparts a force of $2 \cos(5t)$ (in N). Solve for the forced response $U(t)$ in m, expressing $U(t)$ in the form $R \cos(\omega t - \delta)$. Approximate values to 5 decimal places. Hint: be careful with units.

2. [4 parts, 8 points each] Compute the following.

(a) $\mathcal{L}\{t(t+1)\}$

(c) $\mathcal{L}^{-1}\left\{\frac{6}{(s-7)^5}\right\}$

(b) $\mathcal{L}\{2 \cosh(3t) - u_5(t)(t+1)\}$

(d) $\mathcal{L}^{-1}\left\{\frac{s^2}{(s^2+4)(s+2)}\right\}$

3. [20 points] Use the Laplace transform to solve: $y'' - 3y' - 10y = e^t$, $y(0) = 0$ and $y'(0) = 1$.

4. [20 points] Solve the IVP: $y'' + 4y = u_6(t)$, $y(0) = y'(0) = 0$.

5. [4 points] Compute $\mathcal{L}\{1\}$ directly from the definition of the Laplace transform.