Name: _____

Directions: Show all work. No credit for answers without work.

1. [**2 parts, 15 points each**] Solve the following explicitly.

   (a) $\dfrac{dy}{dx} = x - 2y$

   (b) $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2xy}$.

2. [**2 parts, 15 points each**] Solve the following IVPs explicitly.

   (a) $\dfrac{dy}{dt} = 2y^2 - 8$ with $y(0) = -3$.

   (b) $\dfrac{dy}{dt} = \dfrac{\tan t}{y}$ with $y(0) = -1$. *Hint*: to solve $\int \tan t \, dt$ use $\tan t = \frac{\sin t}{\cos t}$ and a substitution.

3. A person wishes to finance a $30,000 car with a loan that has an annual interest rate of 4%. Assume that the loan payment is continuous and interest is compounded continuously.

    (a) [**8 points**] Let $B(t)$ be the balance of the loan (in dollars) at time t (in years), and let k be the annual payment rate. Write a differential equation for $B(t)$.

    (b) [**8 points**] Solve the differential equation.

    (c) [**4 points**] Assuming the loan must be paid in full in 5 years, compute the annual payment rate, and convert it to a monthly rate.

4. [**4 parts, 5 points each**] If possible, apply the existence and uniqueness theorems to the following differential equations. On the basis of these theorems, what can you conclude?

(a) $(\sqrt{t-2})\dfrac{dy}{dt} + (t-6)y = \ln(8-t)$ with $y(3) = 5$

(b) $\dfrac{dy}{dt} = \dfrac{2t}{\cos y}$ with $y(0) = 0$

(c) $\dfrac{dy}{dx} = x\sqrt{y}$ with $y(4) = 0$

(d) $\dfrac{dy}{dx} = y\sqrt{x}$ with $y(4) = 0$