Name: Solutions

1. **[3 points]** Solve the IVP $4y'' + 12y' + 9y = 0$ with $y(0) = 1$ and $y'(0) = 2$.

$$4r^2 + 12r + 9 = 0$$

$$(2r+3)^2 = 0$$

$$r = -\frac{3}{2}, \text{ multiplicity } 2.$$

$$y = c_1 e^{-\frac{3}{2}t} + c_2 t e^{-\frac{3}{2}t}$$

$$y' = -\frac{3}{2} c_1 e^{-\frac{3}{2}t} + c_2 e^{-\frac{3}{2}t} - \frac{3}{2} c_2 t e^{-\frac{3}{2}t}$$

$y(0)=1$: $\left[\, 1 = c_1 \,\right]$

$y'(0)=2$: $2 = -\frac{3}{2}c_1 + c_2 - 0$

$2 = -\frac{3}{2}\cdot 1 + c_2$

$$c_2 = 2 + \frac{3}{2} = \frac{7}{2}$$

$$\boxed{y = e^{-\frac{3}{2}t} + \frac{7}{2} t e^{-\frac{3}{2}t}}$$

2. **[3 points]** Find the general solution to $y^{(4)} + 8y^{(3)} + 17y^{(2)} = 0$.

$$r^4 + 8r^3 + 17r^2 = 0$$

$$r^2(r^2 + 8r + 17) = 0$$

$r = 0$, mult 2,

$$r = \frac{-8 \pm \sqrt{64 - 4 \cdot 17}}{2} = -4 \pm \sqrt{16-17} = -4 \pm i$$

$y_1 = e^{0t} = 1$

$y_2 = te^{0t} = t$

$$y = e^{(-4+i)t} = e^{-4t} \cdot e^{ti} = e^{-4t}(\cos(t) + i\sin(t))$$

$$y_3 = e^{-4t}\cos(t) \qquad y_4 = e^{-4t}\sin(t)$$

So $y = c_1 y_1 + c_2 y_2 + c_3 y_3 + c_4 y_4$

$$y = \boxed{c_1 + c_2 t + c_3 e^{-4t}\cos(t) + c_4 e^{-4t}\sin(t)}$$

3. **[3 points]** Find the general solution to $y'' + 2y' + 5y = \sin t$.

① Gen Soln to $y'' + 2y' + 5y = 0$:

$r^2 + 2r + 5 = 0$

$r = \frac{-2 \pm \sqrt{4 - 4\cdot 5}}{2} = -1 \pm \sqrt{1-5}$

$= -1 \pm 2i$

$y = c_1 e^{-t}\cos(2t) + c_2 e^{-t}\sin(2t)$

② $\sin t$
$\downarrow$
$\cos t$
$\downarrow$
$\cancel{-\sin t}$

Family is $\{\sin t, \cos t\}$.
No conflicts with gen. soln.

$Y = A\sin t + B\cos t$

$Y' = A\cos t - B\sin t$

$Y'' = -A\sin t - B\cos t$

$Y'' + 2Y' + 5Y = \sin t$

$$\begin{aligned} &-A\sin t - B\cos t \\ &+2[-B\sin t + A\cos t] \\ &+5[A\sin t + B\cos t] \end{aligned}$$

$(4A - 2B)\sin t + (4B + 2A)\cos t = \sin t$

$$\begin{aligned} [4A - 2B &= 1]\cdot 2 \\ 2A + 4B &= 0 \end{aligned}$$

$10A = 2$

$A = \frac{1}{5}$, $2B = 4A - 1 = \frac{4}{5} - 1 = -\frac{1}{5}$

$B = -\frac{1}{10}$

$Y = \frac{1}{5}\sin(t) - \frac{1}{10}\cos(t)$

Gen Soln:

$$\boxed{y = c_1 e^{-t}\cos(2t) + c_2 e^{-t}\sin(2t) + \frac{1}{5}\sin(t) - \frac{1}{10}\cos(t)}$$

4. **[1 point]** Given that y_1 is a solution to $y'' + p(t)y' + q(t)y = 0$, the reduction of order procedure looks for additional solutions of the form $y =$ ___$v y_1$___, where $v(t)$ is a function of t.