Name: Solutions

1. **[2 parts, 1 point each]** Compute the following.

(a) $\frac{3+2i}{4-i} \cdot \frac{4+i}{4+i}$

$$= \frac{(3+2i)(4+i)}{(4-i)(4+i)}$$

$$= \frac{12+11i+2i^2}{16-i^2}$$

$$= \frac{10+11i}{17} = \boxed{\frac{10}{17}+\frac{11}{17}i}$$

(b) $(2+i)e^{1-\frac{\pi}{2}i} = (2+i)e \cdot e^{-\pi/2\, i}$

$$= (2+i)e\left[\cos\left(-\tfrac{\pi}{2}\right) + i\sin\left(-\tfrac{\pi}{2}\right)\right]$$

$$= (2+i)e\left[0 + i(-1)\right]$$

$$= -(2+i)ie = -(2i+i^2)e$$

$$= -(2i-1)e = (1-2i)e$$

$$= \boxed{e - 2ei}$$

2. **[3 points]** Using a step size of $h = 0.5$, use Euler's method to approximate $y(0.5)$, $y(1)$, and $y(1.5)$ in the initial value problem with $y' = 2(y - x)$ with $y(0) = 1$.

$(x_0, y_0) = (0, 1)$ $\quad y' = 2(y_0 - x_0) = 2(1-0) = 2$

$x_1 = 0.5, \quad y_1 = y_0 + mh = 1 + 2(0.5) = 2$

$(x_1, y_1) = (0.5, 2) \quad y_2 = y_1 + mh = 2 + \left[2(2-\tfrac{1}{2})\right]\tfrac{1}{2} = 2 + [4-1]\tfrac{1}{2} = 3.5$

$(x_2, y_2) = (1, 3.5) \quad y_3 = y_2 + hm = 3.5 + \tfrac{1}{2}\left[2(3.5-1)\right] = 3.5 + 2.5 = 6$

$(x_3, y_3) = (1.5, 6)$

$$\boxed{\text{So } y(0.5) \approx 2, \quad y(1) \approx 3.5, \quad y(1.5) \approx 6}$$

3. **[2 points]** Indicate whether the given differential equations are linear and separable, or can be so transformed after suitable algebraic manipulation. You do not need to show your work.

Equation	Linear? (Yes/No)	Separable? (Yes/No)
$y' = 3t^2y + t$	Yes	No
$y' = 4y^2 \sin t$	No	Yes
$(3x)dx - (4y)dy = 0$	No	Yes
$(y')^3 = ty$	No	Yes

4. **[3 points]** Find an integrating factor $\mu(x)$ that depends only on x to solve

Note: Same as on worksheet 5!

$$\frac{dy}{dx} = -\left(\frac{y\sin x + 2yx(\cos x)}{x\sin x}\right).$$

Hint: rewrite the equation in standard differential form. After transforming to an exact equation, try imposing $\psi_y = N$ first.

$x\sin x\, dy = -(y\sin x + 2yx(\cos x))\,dx$

$(y\sin x + 2yx\cos x)\,dx + x\sin x\, dy = 0$

- $M_y = \sin x + 2x\cos x$
- $N_x = \sin x + x\cos x$
- $\dfrac{M_y - N_x}{N} = \dfrac{x\cos x}{x\sin x} = \cot x$
- $\mu' = \dfrac{M_y - N_x}{N}\mu$

$\mu' = \mu(\cot x)$

$\int \frac{1}{\mu}\,d\mu = \int \frac{\cos x}{\sin x}\,dx$

$\ln|\mu| = \ln|\sin x| + C$

$\mu = C\sin x$

Choose $\mu = \sin x$

New Eqn:

$\underbrace{\sin x\,(y\sin x + 2yx\cos x)}_{M}\,dx + \underbrace{x\sin^2 x}_{N}\,dy = 0$

Impose $\psi_y = N$: $\psi = \int x\sin^2 x\, dy$

$= x\sin^2 x\, y + h(x)$

Impose $\psi_x = M$:

$\frac{\partial}{\partial x}\left[yx\sin^2 x + h(x)\right] = M$

$y\sin^2 x + yx(2\sin x)(\cos x) + h'(x) = M$

$\cancel{y\sin^2 x} + \cancel{yx(2\sin x)(\cos x)} + h'(x)$

$= \cancel{y\sin^2 x} + \cancel{2yx\cos x\sin x}$

$h'(x) = 0$

$h(x) = C$

So $\psi = x\sin^2 x\, y + C$

Soln:

$\boxed{x\sin^2 x\, y = C}$