

Name: Solutms.

1. (a) [5 points] Find the general solution to
- $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \mathbf{x}$
- .

①
$$\begin{vmatrix} -\lambda & 1 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$-\lambda(1-\lambda) - 2 = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

$$(\lambda - 2)(\lambda + 1) = 0$$

$$\lambda = -1, 2$$

② $\lambda = -1:$

$$\begin{bmatrix} +1 & 1 \\ 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$x_1 + x_2 = 0$$

$$\xi = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

③ $\lambda = 2:$

$$\begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$$

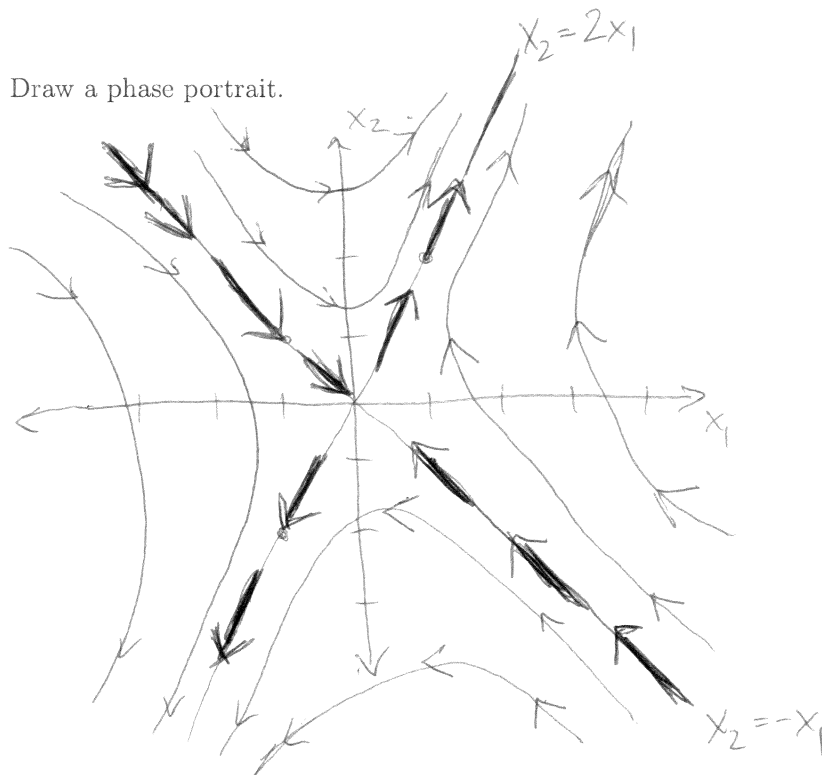
$$-2x_1 + x_2 = 0$$

$$\xi = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

④

$$\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$$

- (b) [2 points] Draw a phase portrait.



2. [3 points] Find all solutions to $z^2 - 4z + (4 + 2i) = 0$.

$$z = \frac{4 \pm \sqrt{16 - 4(4 + 2i)}}{2}$$

$$= 2 \pm \sqrt{\frac{1}{4}(16 - 4(4 + 2i))}$$

$$= 2 \pm \sqrt{4 - (4 + 2i)}$$

$$= 2 \pm \sqrt{-2i}$$

$$= 2 \pm \sqrt{2e^{\frac{3\pi}{2}i}}$$

$$= 2 \pm \sqrt{2} \cdot e^{\frac{3\pi}{4}i}$$

$$= 2 \pm \sqrt{2} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right)$$

$$= 2 \pm \sqrt{2} \left(-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right)$$

$$= 2 \pm (-1 + i)$$

$$z = 2 + (-1 + i) \quad \text{or} \quad z = 2 - (-1 + i)$$

$$= \boxed{1 + i}$$

$$= \boxed{3 - i}$$

