Math261 **Quiz 10** **November 7, 2014**

Name: Solutions

1. **[4 parts, 1.5 points each]** Compute the following.

(a) $\mathcal{L}\left\{2te^{3t}\right\}$

$$= 2\mathcal{L}\{e^{3t}\cdot t\} = 2\mathcal{L}\{t\}\Big|_{s-3} = 2\left(\frac{1}{s^2}\right)\Big|_{s-3}$$

$$= \boxed{\frac{2}{(s-3)^2}}$$

(b) $\mathcal{L}\left\{2\sinh(5t) + 3t^4\right\}$

$$= 2\mathcal{L}\{\sinh(5t)\} + 3\mathcal{L}\{t^4\}$$

$$= 2\cdot\frac{5}{s^2-5^2} + 3\cdot\frac{4!}{s^5} = \boxed{\frac{10}{s^2-25} + \frac{72}{s^5}}$$

(c) $\mathcal{L}^{-1}\left\{\frac{1}{(s-5)^3}\right\}$ $= e^{5t}\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$

$$= \frac{e^{5t}}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} = \frac{e^{5t}}{2}\cdot t^2 = \boxed{\frac{t^2e^{5t}}{2}}$$

(d) $\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2+6s+7}\right\}$ 13 $= \mathcal{L}^{-1}\left\{\frac{2s-1}{(s+3)^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{2(s+3)-7}{(s+3)^2+4}\right\}$

$$= e^{-3t}\mathcal{L}^{-1}\left\{\frac{2s-7}{s^2+4}\right\} = e^{-3t}\left(2\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - \mathcal{L}^{-1}\left\{\frac{7}{s^2+4}\right\}\right)$$

$$= e^{-3t}\left(2\cos(2t) - \frac{7}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}\right) = \boxed{e^{-3t}\left(2\cos(2t) - \frac{7}{2}\sin(2t)\right)}$$

2. **[1 point]** Complete the definition: $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$.

3. **[3 points]** Use the Laplace transform to solve $y'' - 4y' + 5y = 1$ with $y(0) = 1$ and $y'(0) = -1$.

$$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 5\mathcal{L}\{y\} = \mathcal{L}\{1\}$$

$$(s^2Y - sy(0) - y'(0)) - 4(sY - y(0)) + 5Y = \frac{1}{s}$$

$$(s^2Y - s + 1) - 4(sY - 1) + 5Y = \frac{1}{s}$$

$$(s^2 - 4s + 5)Y = \frac{1}{s} + s - 5$$

$$Y = \frac{1}{s^2-4s+5}\left(\frac{1}{s} + \frac{(s-5)s}{s}\right) = \frac{s^2-5s+1}{(s^2-4s+5)s}$$

$$\frac{s^2-5s+1}{s(s^2-4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2-4s+5} \qquad A(s^2-4s+5) + (Bs+C)s = s^2-5s+1$$

$$(A+B)s^2 + (-4A+C)s + 5A = s^2 - 5s + 1$$

$A + B = 1$

$-4A + C = -5$

$5A = 1$

$A = \frac{1}{5}$, $B = \frac{4}{5}$

$C = 4A - 5 = \frac{4}{5} - 5 = \frac{-21}{5}$.

$$Y = \frac{1}{5}\cdot\frac{1}{s} + \frac{1}{5}\cdot\frac{4s-21}{s^2-4s+5}$$

$$= \frac{1}{5}\cdot\frac{1}{s} + \frac{1}{5}\cdot\frac{4(s-2)-13}{(s-2)^2+1}$$

$$= \frac{1}{5}\left(\frac{1}{s} + 4\cdot\frac{s-2}{(s-2)^2+1} - 13\cdot\frac{1}{(s-2)^2+1}\right)$$

$$y = \mathcal{L}^{-1}\{Y\} = \frac{1}{5}\left(\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 4\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2+1}\right\} - 13\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2+1}\right\}\right)$$

$$\boxed{y = \frac{1}{5}\left(1 + 4e^{2t}\cos t - 13e^{2t}\sin(t)\right)}$$