Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 parts, 1 point each]** Differentiate the following functions.

   (a) $y = 3e^{t^2} + \sqrt{t}$

$$y' = 3e^{t^2} \cdot \frac{d}{dt}\left[t^2\right] + \frac{d}{dt}\left[t^{1/2}\right]$$

$$= 6te^{t^2} + \frac{1}{2}t^{-\frac{1}{2}}$$

$$= \boxed{6te^{t^2} + \frac{1}{2\sqrt{t}}}$$

   (b) $y = \frac{e^t - t}{e^t + t}$

$$y' = \frac{(e^t + t)\frac{d}{dt}\left[e^t - t\right] - (e^t - t)\frac{d}{dt}\left[e^t + t\right]}{(e^t + t)^2}$$

$$= \frac{(e^t + t)(e^t - 1) - (e^t - t)(e^t + 1)}{(e^t + t)^2} = \boxed{\frac{2(t-1)e^t}{(e^t + t)^2}}$$

2. **[2 points]** Compute $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ where $w = \sin(xy) + y\ln(x) + e^x$.

$$\frac{\partial w}{\partial x} = y\cos(xy) + \frac{y}{x} + e^x$$

$$\frac{\partial w}{\partial y} = x\cos(xy) + \ln(x)$$

3. [**2 parts, 2 points each**] Solve the following integrals.

(a) $\int te^{t^2}\, dt$

$u = t^2$

$du = 2t\,dt$

$= \frac{1}{2}\int e^{t^2}\, 2t\, dt$

$= \frac{1}{2}\int e^{u}\, du = \frac{1}{2}e^{u} + C = \boxed{\frac{1}{2}e^{t^2} + C}$

(b) $\int \frac{1}{x^2 - 5x + 6}\, dx$

$\frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$, $A(x-3) + B(x-2) = 1$

$\Rightarrow B = 1,\ A = -1$

$= \int \frac{1}{(x-2)(x-3)}\, dx$

$= \int \frac{1}{x-3} - \frac{1}{x-2}\, dx$

$= \ln|x-3| - \ln|x-2| + C = \boxed{\ln\left|\frac{x-3}{x-2}\right| + C}$

(c) $\int \cos^2 t\, dt$

$= \int \frac{1+\cos(2t)}{2}\, dt$

$= \frac{1}{2}\left[\int dt + \int \cos(2t)\, dt\right]$

$= \frac{1}{2}\left[t + \frac{1}{2}\sin(2t)\right] + C$

$= \boxed{\frac{1}{2}t + \frac{1}{4}\sin(2t) + C}$