Directions: Solve the following 6 problems. For computer problems, complete solutions include source code, an **English language overview** of your code, and the final answer. See the course syllabus and the Homework Webpage on the course website for general directions and guidelines.

1. [NT 1-1.5] Prove that $1 + 3 + 5 + \cdots + 2n - 1 = n^2$.

2. [NT 1-1.1] Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

3. Give the base 7 representation for 39201.

4. [NT 1-2.{4,5}]

   (a) Find integers $c_0, \ldots, c_s$ with each $c_s \in \{-1, 0, 1\}$ such that

   $$40189 = c_0 + c_1 \cdot 3 + c_2 \cdot 3^2 + \cdots + c_s 3^s.$$

   (b) Prove that each nonzero integer has a unique representation of the form $c_0 + c_1 \cdot 3 + c_2 \cdot 3^2 + \cdots + c_s 3^s$ with each $c_j \in \{-1, 0, 1\}$ and $c_s \neq 0$.

5. Let $d = \gcd(15708, 1870)$. Find d and obtain integers p and q such that $d = 15708p + 1870q$.

6. Let $A_n = \{(x, y)\colon\ 1 \leq x \leq n,\ 1 \leq y \leq n, \text{ and } \gcd(x, y) = 1\}$. Note that A_n contains all points (x, y) in the $(n \times n)$-grid with corners $(1, 1)$ and (n, n) such that the line segment joining $(0, 0)$ and (x, y) contains no other integer lattice points. The first few such sets are as follows: $A_1 = \{(1, 1)\}$, $A_2 = \{(1, 1), (2, 1), (1, 2)\}$, and $A_3 = \{(1, 1), (1, 2), (1, 3), (2, 3), (2, 1), (3, 1), (3, 2)\}$. Let $f(n) = |A_n|$. Note that $f(1) = 1$, $f(2) = 3$, $f(3) = 7$, and $f(4) = 11$.

   (a) Using a program, compute $f(500)$.

   (b) **[Challenge]** Prove that there is a positive constant C such that $f(n) \geq Cn^2$.