1. [EC 12.1.2] If $R = [-1, 3] \times [0, 2]$, use a Riemann sum with $m = 4$, $n = 2$ to estimate the value of $\iint_R (y^2 - 2x^2)\, dA$. Take sample points to be the upper left corners of the squares.

2. [EC 12.1.{12,16,20}] Calculate the iterated integral.

   (a) $\int_2^4 \int_{-1}^1 (x^2 + y^2)\, dy\, dx$

   (b) $\int_0^1 \int_1^2 \frac{xe^x}{y}\, dy\, dx$

   (c) $\int_0^1 \int_0^1 xy\sqrt{x^2 + y^2}\, dy\, dx$

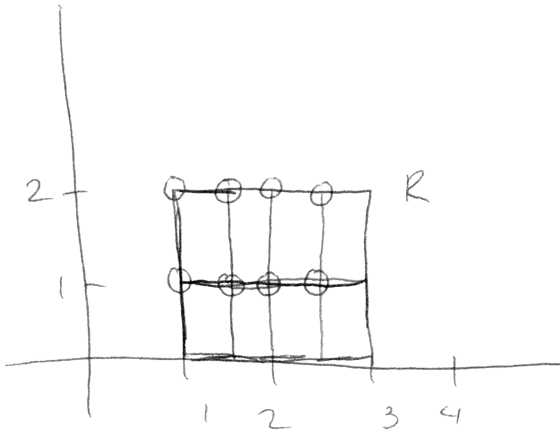
3. [EC 12.1.22] Calculate $\displaystyle\iint_R \cos(x + 2y)\, dA$ for $R = [0, \pi] \times [0, \pi/2]$.

4. [EC 12.2.8] Evaluate the double integral $\displaystyle\iint_D \frac{4y}{x^3 + 2}\, dA$ where $D = \{(x, y) \colon 1 \le x \le 2, 0 \le y \le 2x\}$.

5. [EC 12.2.24] Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, and $z = 0$ in the first octant (where x, y, and z are all at least 0).

Solutions

1.



- 4x2 subgrid.
- Each small rectangle has area $\frac{1}{2} \cdot 1 = \frac{1}{2}$

$$\iint_R y^2 - 2x^2 \, dA \approx \frac{1}{2}\Big(f(1,1) + f(\tfrac{3}{2},1) + f(\tfrac{4}{2},1) + f(\tfrac{5}{2},1)$$

$$+ f(\tfrac{2}{2},2) + f(\tfrac{3}{2},2) + f(\tfrac{4}{2},2) + f(\tfrac{5}{2},2)\Big)$$

$$= \frac{1}{2}\Big(-1 - \frac{7}{2} - 7 - \frac{23}{2} + 2 + (4 - 2\tfrac{9}{4}) + (4-8) + (4 - 2(\tfrac{5}{2})^2)\Big)$$

$$= \frac{1}{2}\Big(-1 - \frac{7}{2} - 7 - \frac{23}{2} + 2 + 4 - \frac{9}{2} - 4 + 4 - \frac{25}{2}\Big)$$

$$= \boxed{-17}$$

2a. $$\int_2^4 \left[\int_{-1}^1 x^2+y^2\, dy\right] dx$$

$$= \int_2^4 \left[\left(x^2y+\frac{y^3}{3}\right)\Big|_{y=-1}^{y=1}\right] dx$$

$$= \int_2^4 \left(x^2+\frac{1}{3}\right)-\left(-x^2+\frac{(-1)^3}{3}\right) dx$$

$$= \int_2^4 2x^2+\frac{2}{3}\, dx = \frac{2}{3}x^3+\frac{2}{3}x\Big|_{x=2}^{x=4}$$

$$= \left(\frac{2}{3}4^3+\frac{2}{3}4\right)-\left(\frac{2}{3}2^3+\frac{2}{3}\cdot 2\right)$$

$$= \frac{2}{3}\left[64+4-8-2\right] = \frac{2}{3}\cdot 58 = \boxed{\frac{116}{3}}$$

b. $$\int_0^1\int_1^2 xe^x\cdot\frac{1}{y}\, dy\, dx = \left[\int_0^1 xe^x\, dx\right]\left[\int_1^2 \frac{1}{y}\, dy\right]$$

$$= \left((x-1)e^x\right)\Big|_{x=0}^{x=1}\left(\ln|y|\right)\Big|_{y=1}^{y=2}$$

$$= \left(0-(-1)e^0\right)\left(\ln|2|-\ln|1|\right)$$

$$= 1\cdot\left(\ln(2)-0\right) = \boxed{\ln(2)}$$

c.

$$\int_0^1 \int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx$$

$$= \int_0^1 x\left[\int_0^1 y\sqrt{x^2+y^2}\,dy\right]dx$$

$u = x^2+y^2$
$du = 2y\,dy$

$$= \int_0^1 x\left[\int_{x^2}^{x^2+1} \sqrt{u}\cdot\tfrac{1}{2}\,du\right]dx$$

$$= \int_0^1 \tfrac{1}{2}x\left(\tfrac{2}{3}u^{3/2}\right)\Big|_{u=x^2}^{u=x^2+1} dx$$

$$= \int_0^1 \tfrac{1}{2}x\left(\tfrac{2}{3}(x^2+1)^{3/2} - \tfrac{2}{3}(x^2)^{3/2}\right)dx$$

$$= \frac{1}{3}\int_0^1 x(x^2+1)^{3/2} - x^4\,dx$$

$$= \frac{1}{3}\int_0^1 x(x^2+1)^{3/2}\,dx - \frac{1}{3}\int_0^1 x^4\,dx$$

$v = x^2+1$
$dv = 2x\,dx$

$$= \frac{1}{3}\int_1^2 v^{3/2}\,\tfrac{1}{2}\,dv - \frac{1}{3}\left(\frac{x^5}{5}\right)\Big|_{x=0}^{x=1}$$

$$= \frac{1}{6}\left(\tfrac{2}{5}v^{5/2}\right)\Big|_{v=1}^{v=2} - \frac{1}{3}\left(\frac{1}{5}-0\right)$$

$$= \frac{1}{15} 2^{5/2} - \frac{1}{15} \cdot 1 - \frac{1}{15}$$

$$= \frac{4\sqrt{2}}{15} - \frac{2}{15} = \boxed{\frac{4\sqrt{2} - 2}{15}}$$

3. $$\iint_R \cos(x+2y)\, dA = \int_0^{\pi} \left[\int_0^{\pi/2} \cos(x+2y)\, dy \right] dx$$

$$= \int_0^{\pi} \left(\frac{1}{2} \sin(x+2y) \right) \Big|_{y=0}^{y=\pi/2} dx$$

$$= \frac{1}{2} \int_0^{\pi} \sin(x+\pi) - \sin(x)\, dx$$

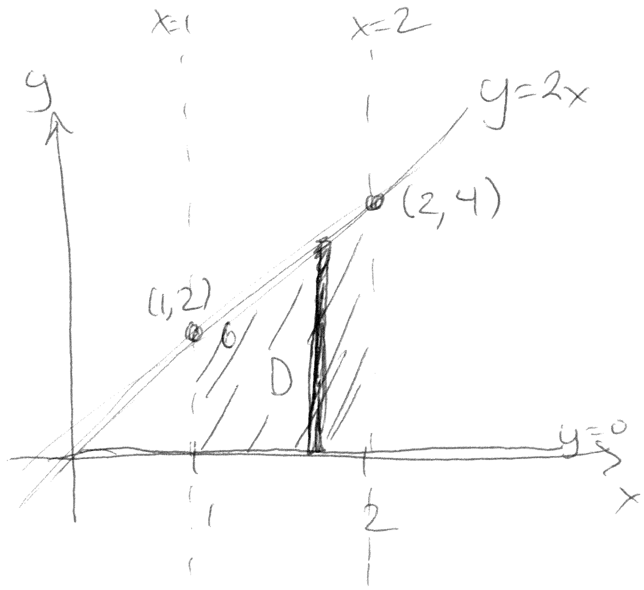
$$= \frac{1}{2} \int_0^{\pi} -\sin(x) - \sin(x)\, dx$$

$$= \int_0^{\pi} -\sin(x)\, dx$$

$$= \cos(x) \Big|_{x=0}^{x=\pi} = \cos(\pi) - \cos(0) = \boxed{-2}$$

⑤

4.



• view D as type 1

$$\iint_D \frac{4y}{x^3+2}\,dA = \int_1^2 \int_0^{2x} \frac{4y}{x^3+2}\,dy\,dx$$

$$= \int_1^2 \left. \frac{2y^2}{x^3+2} \right|_{y=0}^{y=2x} dx$$

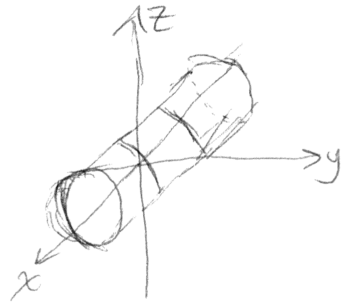
$$= \int_1^2 \frac{8x^2}{x^3+2} - 0\,dx \qquad \begin{aligned} u &= x^3+2 \\ du &= 3x^2\,dx \end{aligned}$$

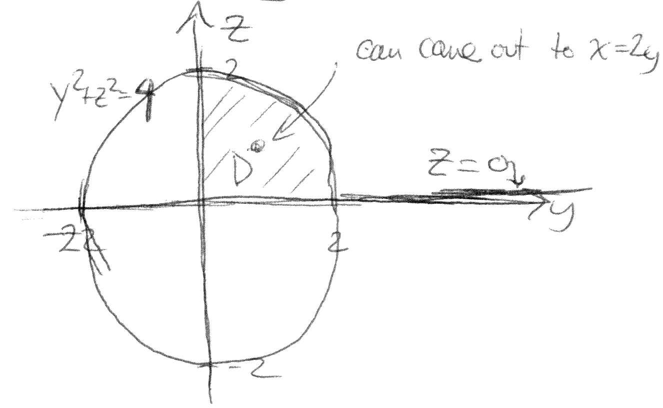
$$= \int_3^{10} \frac{8}{u} \cdot \frac{1}{3}\,du$$

$$= \left(\frac{8}{3} \ln|u| \right)_{u=3}^{u=10} = \frac{8}{3}\ln(10) - \frac{8}{3}\ln(3)$$

$$= \boxed{\frac{8}{3}\ln\left(\frac{10}{3}\right)}$$

5. The cylinder $y^2+z^2=4$ has same cross section at all planes $x=k$:





Set up 1:
BASE Area in $x=0$ plane

$$\iint_D 2y\, dA = \int_0^2 \int_0^{\sqrt{4-y^2}} 2y \; dz \; dy$$

$$= \int_0^2 2yz \Big|_{z=0}^{z=\sqrt{4-y^2}} dy$$

$$= \int_0^2 2y\sqrt{4-y^2}\, dy \qquad u=4-y^2 \quad du=-2y\,dy$$

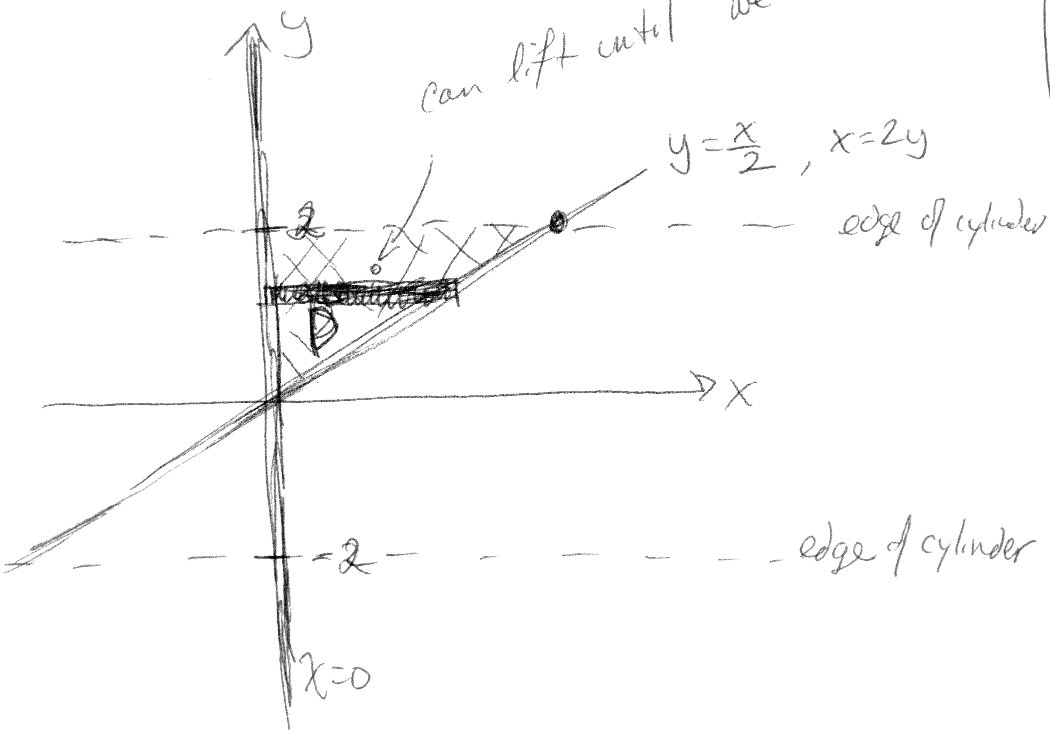
$$= \int_4^0 \sqrt{u}\,(-du)$$

$$= \int_0^4 u^{1/2}\, du = \frac{2}{3}u^{3/2}\Big|_{u=0}^{u=4} = \frac{2}{3}(4)^{3/2} = \frac{2}{3}(2)^3 = \boxed{\frac{16}{3}}$$

Alt. Soln. Base area in $z=0$ plane.

$z=0$ plane:

can lift until we hit cylinder $z^2+y^2=4$, or $z=\sqrt{4-y^2}$



Cylinder
- $y^2+z^2=4$ becomes

$y^2=4$

$y=\pm 2$

- View D as a region of type 2!

$$\iint_D \sqrt{4-y^2}\,dA = \int_0^2\int_0^{2y}\sqrt{4-y^2}\,dx\,dy$$

$$= \int_0^2 \left(\sqrt{4-y^2}\,x\right)\Big|_{x=0}^{x=2y} dy$$

$$= \int_0^2 2y\sqrt{4-y^2}\,dy \qquad u=4-y^2 \quad du=-2y\,dy$$

$$= \int_4^0 -\sqrt{u}\,du = \int_0^4 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_{u=0}^{u=4} = \frac{2}{3}\cdot 4^{3/2}$$

$$= \boxed{\frac{16}{3}}$$