1. [EC 11.5.{2,6,20}] Use the chain rule to find the indicated (partial) derivative(s).

   (a) dz/dt for $z = x\ln(x+2y)$, $x = \sin t$, $y = \cos t$.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

$$= \left(\ln(x+2y) + x\cdot\frac{1}{x+2y}\right)\cdot\cos t + \frac{2x}{x+2y}(-\cancel{\cos}\sin t)$$

$$= \ln(x+2y)\cos t + \frac{x\cos t}{x+2y} - \frac{\cancel{8}2x\sin t}{x+2y}$$

$$= \boxed{\ln(\sin t+2\cos t)(\cos t) + \frac{\sin t\cos t}{\sin t+2\cos t} - \frac{2(\sin t)^2}{\sin t+2\cos t}}$$

   (b) $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for $z = x/y$, $x = se^t$, $y = 1 + se^{-t}$.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = \frac{1}{y}\cdot e^t + -\frac{x}{y^2}\cdot e^{-t}$$

$$= \frac{e^t}{1+se^{-t}} + -\frac{se^t\cdot e^{-t}}{(1+se^{-t})^2} = \frac{e^t}{1+se^{-t}} - \frac{s}{(1+se^{-t})^2} = \boxed{\frac{e^t}{(1+se^{-t})}}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} = \frac{1}{y}\cdot se^t \cancel{+} + \frac{x}{y^2}\cdot se^{-t} = \frac{s}{y}\left(e^t + \frac{se^t}{1+se^{-t}}\cdot e^{-t}\right)$$

$$= \frac{s}{y}\left(\frac{e^t + \cancel{s} + \cancel{s}}{1+se^{-t}}\right) = \boxed{\frac{se^t + 2s^2}{(1+se^{-t})^2}}$$

   (c) $\frac{\partial M}{\partial u}$ and $\frac{\partial M}{\partial v}$ when $(u,v) = (3,-1)$ for $M = xe^{y-z^2}$, $x = 2uv$, $y = u - v$, $z = u + v$.

when $(u,v) = (3,-1)$, $(x,y,z) = (-6,4,2)$.

$$\frac{\partial M}{\partial u} = \frac{\partial M}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial M}{\partial y}\cdot\frac{\partial y}{\partial u} + \frac{\partial M}{\partial z}\cdot\frac{\partial z}{\partial u} = (e^{y-z^2})\cdot(2v) + (xe^{y-z^2})(1) + (-2zxe^{y-z^2})\cdot 1$$

$$= e^{y-z^2}(2v + x - 2zx) = e^{u-v-(u+v)^2}(2v + 2uv - 2(u+v)\cdot 2uv)$$

$$= \boxed{2ve^{u-v-(u+v)^2}(1 + u - 2(u+v)u)}$$

$$\frac{\partial M}{\partial v} = \frac{\partial M}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial M}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial M}{\partial z}\frac{\partial z}{\partial v} = (e^{y-z^2})(2u) + (xe^{y-z^2})(-1) + (-2zxe^{y-z^2})(1)$$

$$= e^{y-z^2}(2u - x - 2zx) = e^{u-v-(u+v)^2}(2u - 2uv - 4(u+v)uv)$$

$$= \boxed{2ue^{u-v-(u+v)^2}(1 - v - 2(u+v)v)}$$

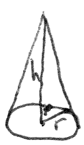
2. [EC 11.5.28] Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $yz = \ln(x+z)$. Implicit.

$$F(x,y,z) = yz - \ln(x+z)$$

- $$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = +\frac{+\frac{1}{x+z}}{y-\frac{1}{x+z}} \cdot \frac{x+z}{x+z} = \frac{1}{y(x+z)-1} = \boxed{\frac{1}{yx+yz-1}}$$

- $$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{z-0}{y-\frac{1}{x+z}} \cdot \frac{x+z}{x+z} = -\frac{z(x+z)}{y(x+z)-1} = \boxed{-\frac{zx+z^2}{yx+yz-1}}$$

3. [EC 11.5.32] The radius of a right circular cone is increasing at a rate of 1.8 in/s while its height is decreasing at a rate of 2.5 in/s. At what rate is the volume of the cone changing when the radius is 120 in. and the height is 140 in.?



$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\cdot\frac{dr}{dt} + \frac{\partial V}{\partial h}\cdot\frac{dh}{dt} = \left(\frac{2}{3}\pi h r\right)\cdot(1.8) + \left(\frac{1}{3}\pi r^2\right)(-2.5)$$

$$= \frac{\pi r}{3}\left[3.6h - 2.5r\right] = \frac{\pi}{3}\cdot 120\cdot\left[(3.6)(140) - (2.5)(120)\right]$$

$$\approx \boxed{25{,}635.4 \ \text{in}^3/\text{s}.}$$

4. [EC 11.5.44(b,c)] Find $\partial z/\partial\theta$ and $\partial^2 z/\partial r\partial\theta$ for $z = f(x,y)$ where $x = r\cos\theta$ and $y = r\sin\theta$.

- $$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = f_x\cdot(-r\sin\theta) + f_y\cdot(r\cos\theta) = \boxed{r\left(f_y\cos\theta - f_x\sin\theta\right)}$$

- $$\frac{\partial^2 z}{\partial r\partial\theta} = \frac{\partial}{\partial r}\left[\frac{\partial z}{\partial \theta}\right] = \frac{\partial}{\partial r}\left[r\left(f_y\cos\theta - f_x\sin\theta\right)\right]$$

$$= \cos\theta\frac{\partial}{\partial r}\left[rf_y\right] - \sin\theta\frac{\partial}{\partial r}\left[rf_x\right]$$

$$= \cos\theta\left(f_y + r\frac{\partial}{\partial r}[f_y]\right) - \sin\theta\left(f_x + r\frac{\partial}{\partial r}[f_x]\right)$$

$$= \cos\theta\left(f_y + r\left(\frac{\partial f_y}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f_y}{\partial y}\frac{\partial y}{\partial r}\right)\right) - \sin\theta\left(f_x + r\left(\frac{\partial f_x}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f_x}{\partial y}\frac{\partial y}{\partial r}\right)\right)$$

$$= \boxed{\cos\theta\left(f_y + r\left(f_{yx}\cos\theta + f_{yy}\sin\theta\right)\right) - \sin\theta\left(f_x + r\left(f_{xx}\cos\theta + f_{xy}\sin\theta\right)\right)}$$