Solutions

Recall For any const a: FTC: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

1. [EC 11.3.{10,12,18,22}] Find the first partial derivatives of the function.

(a) $z = y \ln x$

$$\boxed{\frac{\partial z}{\partial x} = y \cdot \frac{1}{x}}$$

$$\boxed{\frac{\partial z}{\partial y} = \ln x}$$

(b) $f(x, y) = x^y$

$$\boxed{f_x = y x^{y-1}}$$

Since $x^y = e^{\ln(x) y}$

$$f_y = (\ln x)' \cdot e^{\ln(x) y}$$

$$= \boxed{(\ln x) \cdot x^y}$$

(c) $f(x, y) = \int_y^x \cos(t^2)\, dt$

$$f_x = \boxed{\cos(x^2)}$$

$$f_y = \frac{\partial}{\partial y}\left[\int_y^x \cos(t^2)\,dt\right]$$

$$= \frac{\partial}{\partial y}\left[-\int_x^y \cos(t^2)\,dt\right] = \boxed{-\cos(y^2)}$$

(d) $w = \sqrt{r^2 + s^2 + t^2}$

- $$\frac{\partial w}{\partial r} = \frac{1}{2}(r^2+s^2+t^2)^{-\frac{1}{2}} \cancel{2}r$$

$$= \boxed{\frac{r}{\sqrt{r^2+s^2+t^2}}}.$$

- $$\boxed{\frac{\partial w}{\partial s} = \frac{s}{\sqrt{r^2+s^2+t^2}}, \quad \frac{\partial w}{\partial t} = \frac{t}{\sqrt{r^2+s^2+t^2}}}$$

2. [EC 11.3.38] Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $yz = \ln(x + z)$.

- $$\frac{\partial z}{\partial x}[yz] = \frac{\partial z}{\partial x}[\ln(x+z)]$$

$$y\frac{\partial z}{\partial x} = \frac{1}{x+z}\left(1 + \frac{\partial z}{\partial x}\right)$$

$$\left(y - \frac{1}{x+z}\right)\frac{\partial z}{\partial x} = \frac{1}{x+z}$$

$$\frac{\partial z}{\partial x} = \frac{1}{x+z} \cdot \frac{x+z}{yx+yz-1} = \boxed{\frac{1}{yx+yz-1}}$$

- $$\frac{\partial z}{\partial y}[yz] = \frac{\partial z}{\partial y}[\ln(x+z)]$$

$$z + y\frac{\partial z}{\partial y} = \frac{1}{x+z}\frac{\partial z}{\partial y}$$

$$\left(y - \frac{1}{x+z}\right)\frac{\partial z}{\partial y} = -z$$

$$\frac{\partial z}{\partial y} = \boxed{-z \cdot \frac{x+z}{yx+yz-1}}$$

3. [EC 11.3.44] Find all four second partial derivatives of $f(x, y) = \ln(3x + 5y)$.

$f_x = \frac{1}{3x+5y} \cdot 3 = 3(3x+5y)^{-1}$, $\quad f_y = \frac{1}{3x+5y} \cdot 5 = 5(3x+5y)^{-1}$

Note: $f_{xy} = f_{yx}$ by Clairaut's Thm.

- $f_{xx} = -3(3x+5y)^{-2} \cdot 3 = \boxed{\frac{-9}{(3x+5y)^2}}$
- $f_{xy} = -3(3x+5y)^{-2} \cdot 5 = \boxed{\frac{-15}{(3x+5y)^2}}$
- $f_{yx} = -5(3x+5y)^{-2} \cdot 3 = \boxed{\frac{-15}{(3x+5y)^2}}$
- $f_{yy} = -5(3x+5y)^{-2} \cdot 5 = \boxed{\frac{-25}{(3x+5y)^2}}$

4. [EC 11.4.4] Find the equation of the tangent plane to $z = y \ln x$ at $(1, 4, 0)$.

- $f_x = \frac{y}{x}$; $\quad f_x(1,4) = \frac{4}{1} = 4$. $\quad$ - $f_y = \ln x$, $\quad f_y(1,4) = \ln 1 = 0$

So
$$z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$z - 0 = 4(x-1) + 0(y-4)$$
$$\boxed{z = 4x - 4}$$

5. [EC 11.4.30] The pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12 L to 12.3 L and the temperature decreases from 310 K to 305 K.

$$P = 8.31 \frac{T}{V}$$

$$dP = \frac{\partial}{\partial T}[P]\, dT + \frac{\partial}{\partial V}[P]\, dV$$
$$= \frac{8.31}{V}\, dT + \left(\frac{-(8.31)T}{V^2}\right) dV$$

- $\Delta P \approx dP = \frac{8.31}{12}(-5) - \frac{(8.31)(310)}{(12)^2} \cdot (0.3) \approx \boxed{-8.83 \text{ kPa}}$

- So, pressure decreases by approximately $\boxed{8.83 \text{ kPa}}$