Name: Solutions

Directions: Solve all problems.

1. [EC 11.2.{4-16} even]. In (a)-(g), find the limit if it exists, or show that it does not exist.

(a) $\lim_{(x,y)\to(6,3)} xy\cos(x-2y)$

Since $xy\cos(x-2y)$ is cont. on $\mathbb{R}^2$:

$= (6)(3)\cos(6-2\cdot 3)$

$= 18\cos(0)$

$= \boxed{18}$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2}$

With $y=0$:

$\lim_{x\to 0} \frac{x^2+\sin^2 0}{2x^2+0^2}$

$= \lim_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$

With $x=0$

$\lim_{y\to 0} \frac{0^2+\sin^2 y}{0+y^2}$

$= \lim_{y\to 0} \frac{\sin y}{y}\cdot\frac{\sin y}{y} = 1$

$\boxed{\text{Limit DNE}}$

(c) $\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$

- Along $y=0$:

$\lim_{x\to 0} \frac{6x^3(0)}{2x^4+0} = \frac{0}{2x^4} = 0$

- Along $y=x$:

$\lim_{x\to 0} \frac{6x^3(x)}{2x^4+x^4} = \lim_{x\to 0}\frac{6x^4}{3x^4} = 2$

So $\boxed{\text{limit DNE}}$

(d) $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2}$

Since $0\le x^2\le x^2+2y^2$, $\left|\frac{x^2}{x^2+2y^2}\right|\le 1$.

So $\left|\frac{x^2\sin^2 y}{x^2+2y^2}\right| = \left|\frac{x^2}{x^2+2y^2}\right|\cdot|\sin^2 y|$

$\le |\sin^2 y|$.

$-\sin^2 y \le \frac{x^2\sin^2 y}{x^2+2y^2} \le \sin^2 y$

Hence $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2} = \boxed{0}$ by Squeeze theorem.

(e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$

Along $y=0$

$$\lim_{x\to 0} \frac{x\cdot 0}{x^2+0} = \lim_{x\to 0} \frac{0}{x^2} = 0$$

Along $x=y^4$:

$$\lim_{y\to 0} \frac{(y^4)y^4}{(y^4)^2+y^8} = \lim_{y\to 0} \frac{y^8}{2y^8} = \frac{1}{2}$$

So $\boxed{\text{limit DNE}}$.

(f) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{\cancel{(x^2+y^2)}(x^2-y^2)}{\cancel{x^2+y^2}}$$

$$= \lim_{(x,y)\to(0,0)} x^2-y^2$$

$$= \boxed{0}.$$

(g) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^2+2y^2+3z^2}{x^2+y^2+z^2}$

Along $y=z=0$:

$$\lim_{x\to 0} \frac{x^2+0+0}{x^2+0+0} = 1$$

Along $x=y=0$:

$$\lim_{z\to 0} \frac{0+0+3z^2}{0+0+z^2} = 3$$

So $\boxed{\text{limit DNE.}}$

(h) Determine the set of points at which $F(x,y) = \frac{x-y}{1+x^2+y^2}$ is continuous.

$F(x,y)$ is a ratio of continuous functions, so F is continuous on it's domain. Since $1+x^2+y^2 \geq 1$, the denom. is never 0 and $\boxed{F \text{ is continuous on } \mathbb{R}^2.}$