Name: _____

Directions: Show all work. No credit for answers without work. This test has 100 points but scores will be taken out of 88.

1. [**12 points**] Let $f(x, y) = \ln(y/x^2)$. (i) What is the domain of f? (ii) Sketch a contour map of f showing the level curves of heights 0 and 1. Label each curve with its height.

2. [**2 parts, 6 points each**] Find the limit, if it exists, or show that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + xy}{x^2 + y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$

3. [**4 points**] Let $f(x, y)$ be a differentiable function. Express $f_y(2, 3)$ as the limit of a difference quotient.

4. [**12 points**] Let $f(x, y) = xe^{x^2 y}$. Find f_x and f_y.

5. [**12 points**] Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ where $xyz^2 + x\cos(z) = 0$.

6. [**12 points**] Find the equation of the tangent plane to the surface of $z = \sin(x^2 - y^2)$ at the point $(1, 1, 0)$.

7. [**12 points**] The width w and height h of a rectangle are changing. At a particular instant in time, $w = 5$ cm and increasing at 2 cm/s, and $h = 8$ cm and decreasing at 3 cm/s. Find the rate of change in (i) the area of the rectangle, and (ii) the length of its diagonal.

8. [**12 points**] Find the directional derivative of $f(x, y) = y^2 \tan(x)$ at $(\pi/4, -2)$ in the direction of $2\vec{i} + \vec{j}$.

9. [**12 points**] Find and classify the critical points of $f(x, y) = x^3 y + 12x^2 - 8y$ as local minimums, local maximums, or saddle points.