Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. **[6 points]** Let V be the vector space of real-valued functions on the whole real line, over the field of real numbers. Is $\{x^2, x^2+3x, 2x\}$ linearly independent in V? Justify your answer.

2. **[3 parts, 2 points each]** The following questions are about the vector space $\mathbb{R}^4$.

   (a) Is there a vector $\vec{a}$ in $\mathbb{R}^4$ that is contained in the span of every set of vectors? If so, write down $\vec{a}$ explicitly. If not, explain why not.

   (b) Let S be a set of vectors in $\mathbb{R}^4$. Is it true that if $\vec{a} \in S$, then $\vec{a}$ is in the span of S? Explain.

   (c) Give an example of a basis of $\mathbb{R}^4$.

3. **[6 parts, 2 points each]** Let

$$A = \begin{bmatrix} 3 & 1+i & 2-i \\ 4 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 0 \\ 1 & i \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad D = \begin{bmatrix} 3+2i & -7 \\ 0 & 1 \\ 2i & 0 \end{bmatrix}$$

be matrices over the field of complex numbers $\mathbb{C}$. For each of the following, write the specified matrix explicitly if possible, or write "undefined" otherwise.

(a) $A + B$

(b) iD

(c) $\overline{A}$

(d) C^T

(e) AA

(f) BB

4. **[12 points]** Using matrices and Gauss-Jordan elimination, find all solutions to the following system of linear equations.

$$\begin{array}{rcrcrcrcl}
x_1 & + & 2x_2 & - & 2x_3 & + & 6x_4 & = & 0 \\
 & & 2x_2 & + & 4x_3 & - & 2x_4 & = & 0 \\
2x_1 & & & - & 12x_3 & + & 16x_4 & = & 0 \\
-2x_1 & + & 3x_2 & + & 18x_3 & - & 19x_4 & = & 0
\end{array}$$

5. **[6 points]** Find a matrix in Reduced Row Echelon Form that is row-equivalent to the matrix A below.

$$A = \begin{bmatrix} 3 & 15 & -2 & 11 & 2 \\ 2 & 10 & -3 & 9 & -2 \end{bmatrix}$$

6. **[6 points]** Consider the vector space $\mathbb{R}^3$ and let $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$. Which vectors $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ are in span(S)? Give a simple condition on b_1, b_2, and b_3 which answers this question.