Math251 | Quiz 6 | October 16, 2013

Name: Solutions

Directions: Show all work. No credit for answers without work.

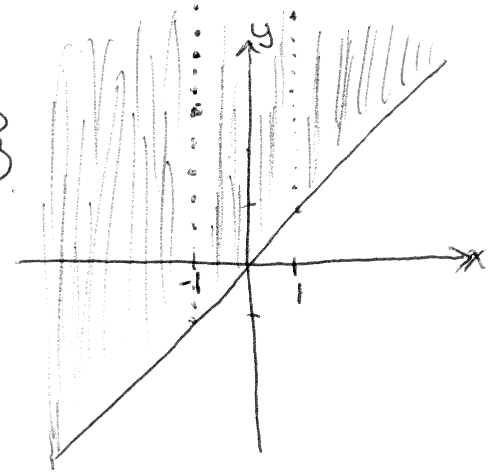
1. **[2 points]** Find and sketch the domain of the function $f(x,y) = \dfrac{\sqrt{y-x}}{1-x^2}$.

We need $1-x^2 \neq 0$ and $y-x \geq 0$. ~~So, $x^2 \neq 1$ and~~

Since $1-x^2=0$ means $(1+x)(1-x)=0$, or $x=1$ or $x=-1$,

we have

$$D = \{(x,y):\ x \neq \pm 1 \text{ and } y \geq x\}$$



2. **[2 points]** Draw a contour map of $f(x,y) = e^{y/x}$ showing four level curves. *Label each level curve with its height.*

Curve at height k:

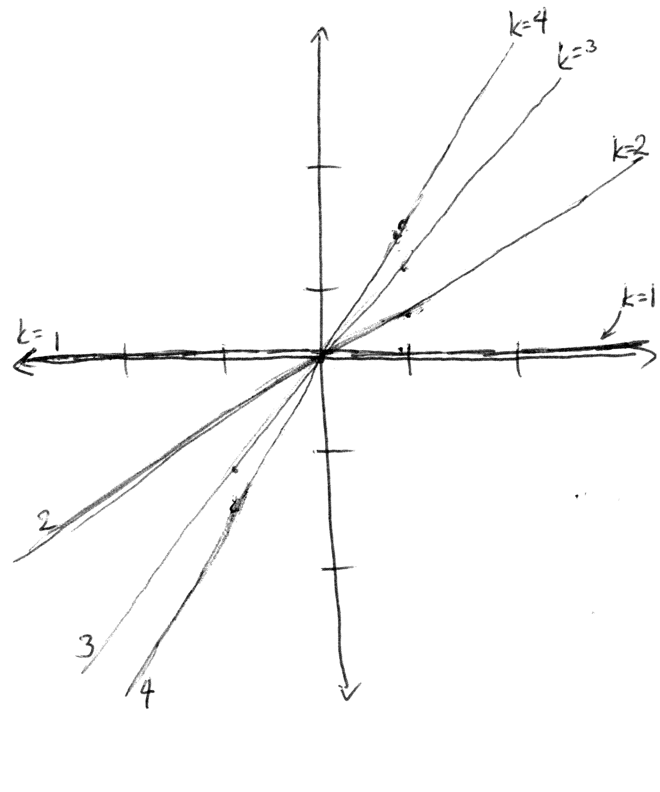
$$e^{y/x} = k$$

$$\frac{y}{x} = \ln k$$

$$\boxed{y = (\ln k)\, x}$$

This is a line through the origin with slope $\ln k$

level k	1	2	3	4
Slope $\ln k$	0	0.69	1.10	1.39



3. **[2 parts, 2 points each]** Find the limit, if it exists, or show the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$

Since $0 \le x^2 \le x^2+y^2$, we have $\left|\dfrac{x^2}{x^2+y^2}\right| \le 1$.

Therefore $\left|\dfrac{x^2y^2}{x^2+y^2}\right| = \left|\dfrac{x^2}{x^2+y^2}\right| |y^2| \le |y^2|$ and

$-y^2 \le \dfrac{x^2y^2}{x^2+y^2} \le y^2$. Since $-y^2$ and y^2 go to 0 as $(x,y)\to(0,0)$, this limit has value $\boxed{0}$ by squeeze thm

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

- Along $y=0$: $\displaystyle\lim_{x\to0}\frac{(x^2)(0)}{x^4+(0)^2} = \lim_{x\to0}\frac{0}{x^4} = 0$

- Along $y=x^2$: $\displaystyle\lim_{x\to0}\frac{(x^2)(x^2)}{x^4+(x^2)^2} = \lim_{x\to0}\frac{x^4}{2x^4} = \frac{1}{2}$

Since these approaches have different limits, $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2y}{x^4+y^2}$ $\boxed{\text{DNE.}}$

4. **[2 points]** Determine the set of points at which the function $f(x,y) = \dfrac{x+y}{x^2-y^2}$ is continuous.

Since $f(x,y)$ is rational, it is continuous wherever ~~x+y~~ it is defined and $x+y$ and x^2-y^2 are continuous.

- Since $x+y$ and x^2-y^2 are polynomials, these are continuous on $\mathbb{R}^2$

- Therefore f is continuous on its domain

$$\{(x,y) \mid x^2-y^2 \ne 0\} = \boxed{\{(x,y) \mid y \ne \pm x\}}$$