Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 points]** Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

$$(\text{Radius})^2 = \cancel{\sqrt{}}(4-3)^2 + (3-8)^2 + (-1-1)^2 = 1 + 25 + 4 = 30$$

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

$$\boxed{(x-3)^2 + (y-8)^2 + (z-1)^2 = 30}$$

2. **[2 points]** Find the angle between $2\vec{i} - 3\vec{j} + 5\vec{k}$ and $4\vec{i} + 2\vec{j} + 1\vec{k}$ in radians and degrees.

$$\vec{a} = \langle 2, -3, 5 \rangle, \quad \vec{b} = \langle 4, 2, +1 \rangle$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{(2)(4) + (-3)(2) + (5)(+1)}{(\sqrt{4+9+25})(\sqrt{16+4+1})} = \frac{7}{\sqrt{38\cdot 21}} = \frac{7}{\sqrt{798}}$$

$$\text{So } \theta = \cos^{-1}\left(\frac{7}{\sqrt{798}}\right) \approx \boxed{1.32 \text{ rad}} \approx 1.32\,\frac{180}{\pi} \approx \boxed{75.65^\circ}$$

3. **[2 points]** Which of the following expressions are meaningful, and which are meaningless? **Circle** the expressions that are **meaningful**. Here, $\vec{a}$, $\vec{b}$, and $\vec{c}$ are all vectors in $\mathbb{R}^3$.

(a) $(\vec{a}\cdot\vec{b})\cdot\vec{c}$ scalar • vector

(b) $(\vec{a}\cdot\vec{b})\vec{c}$

(c) $|\vec{a}|(\vec{b}\cdot\vec{c})$

(d) $\vec{a}\cdot(\vec{b}+\vec{c})$

(e) $\vec{a}\cdot\vec{b}+\vec{c}$ scalar + vector

(f) $|\vec{a}|\cdot(\vec{b}+\vec{c})$ scalar • vector

not a vector

4. **[2 points]** Find a vector orthogonal to both $\langle 1, -2, 4\rangle$ and $\langle 3, 1, -1\rangle$.

Find cross product: $\vec{a}$ $\vec{b}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 4 \\ 3 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -2 & 4 \\ 1 & -1 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix}\vec{k}$$

$$= (2-4)\vec{i} - (-1-12)\vec{j} + (1-(-6))\vec{k}$$

$$= \boxed{-2\vec{i} + 13\vec{j} + 7\vec{k} = \langle -2, 13, 7\rangle}$$

5. **[2 points]** Find the area of the triangle PQR with vertices $P(2,0,4)$, $Q(1,-1,-2)$, and $R(3,1,5)$.

$$\vec{a} = \overrightarrow{PQ} = \langle 1-2, -1-0, -2-4\rangle = \langle -1, -1, -6\rangle$$

$$\vec{b} = \overrightarrow{PR} = \langle 3-2, 1-0, 5-4\rangle = \langle 1, 1, 1\rangle$$

$$\text{Area} = \frac{1}{2}|\vec{a}\times\vec{b}|. \qquad \vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -1 & -6 \\ 1 & 1 & 1 \end{vmatrix} = (-1-(-6))\vec{i} - (-1-(-6))\vec{j} + ((-1)-(-1))\vec{k}$$

$$= 5\vec{i} - 5\vec{j} + 0\vec{k}$$

$$\text{So Area} = \frac{1}{2}|\langle 5, -5, 0\rangle| = \frac{1}{2}\sqrt{25+25+0} = \frac{1}{2}\sqrt{2\cdot 5^2}.$$

$$= \boxed{\frac{5\sqrt{2}}{2}}$$