

Name: _____

Solutions

Directions: Show all work. No credit for answers without work.

1. [2 points] Write the
- (3×2)
- matrix
- A
- where
- $a_{ij} = 2i - j$
- .

$$\begin{array}{cc}
 & \begin{array}{c} -1 \quad 2 \\ \hline j \end{array} \\
 \begin{array}{c} i \\ \\ \\ \end{array} & \begin{array}{cc} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{array}
 \end{array}$$

2. [1 point] Complete the following sentence: if
- A
- is an
- $(m \times n)$
- matrix, then the product
- AA
- is defined if and only if

$$m = n$$

or

$$A \text{ is square}$$

3. [1 point] True or false: matrix addition is commutative. True
4. [1 point] True or false: matrix multiplication is commutative. False
5. [1 point] Complete the following sentence: matrix multiplication is associative since for all matrices $A, B,$ and C, \dots

$$(AB)C = A(BC)$$

6. [1 point] Explicitly write down a matrix
- X
- in
- $\mathbb{Q}^{3 \times 3}$
- such that for every matrix
- A
- in
- $\mathbb{Q}^{3 \times 3}$
- , the equations
- $AX = A$
- and
- $XA = A$
- hold. What is
- X
- called?

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

X is the (3×3) -identity matrix I_3 .

7. [3 points] Let

$$A = \begin{bmatrix} i & 2-i & 0 \\ 3 & 1 & 4-2i \end{bmatrix} \quad B = \begin{bmatrix} i & 0 \\ 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -i \\ 3 & 1 \end{bmatrix}$$

be matrices over the field of complex numbers \mathbb{C} . For each of the following, write the specified matrix explicitly if possible, or write "undefined" otherwise.

(a) $3B$

$$\begin{bmatrix} 3i & 0 \\ 3 & 6 \end{bmatrix}$$

(b) $B+iC$

$$\begin{bmatrix} 3i & 1 \\ 1+3i & 2+i \end{bmatrix}$$

(c) A^*

$$\begin{bmatrix} -i & 3 \\ 2+i & 1 \\ 0 & 4+2i \end{bmatrix}$$

(d) The additive inverse of C

$$\begin{bmatrix} -2 & i \\ -3 & -1 \end{bmatrix}$$

(e) AB

undefined

(f) BA

$$\begin{bmatrix} i & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} i & 2-i & 0 \\ 3 & 1 & 4-2i \end{bmatrix} \\ = \begin{bmatrix} -1 & 1+2i & 0 \\ 6+i & 4-i & 8-4i \end{bmatrix}$$