Name: Solution

Directions: Show all work. No credit for answers without work.

1. [**2 points**] Find the derivative of $f(x) = 2^{\tan(x)} - \ln(x^5)$.

$$f'(x) = \frac{d}{dx}\left[2^{\tan(x)}\right] - \frac{d}{dx}\left[5\ln(x)\right]$$

$$= \ln(2)\cdot 2^{\tan(x)}\frac{d}{dx}\left[\tan(x)\right] - \frac{5}{x}$$

$$= \boxed{\ln(2)\cdot 2^{\tan(x)}\sec^2(x) - \frac{5}{x}}$$

2. [**3 points**] Evaluate $\int_0^{\pi/2} \sin(x)\cos(x)\,dx$.

Soln 1: $u = \sin(x)$, $du = \cos(x)dx$

$$\int_0^{\pi/2} \underbrace{\sin(x)}_{u}\underbrace{\cos(x)dx}_{du}$$

$$= \int_0^1 u\,du = \frac{u^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$$

Soln 2: $\sin(2x) = 2\sin(x)\cos(x)$, so

$$\int_0^{\pi/2}\sin(x)\cos(x)dx = \frac{1}{2}\int_0^{\pi/2}\sin(2x)dx$$

$$= \frac{1}{2}\left(-\frac{\cos(2x)}{2}\right)\Bigg|_0^{\pi/2}$$

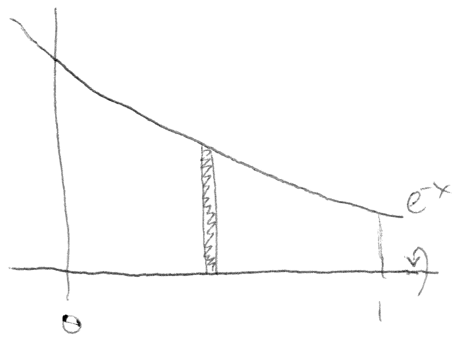
$$= \frac{1}{2}\left(\frac{-\cos(\pi)}{2} + \frac{\cos(0)}{2}\right)$$

$$= \boxed{\frac{1}{2}}$$

3. [**5 points**] Find integrals for the following quantities. *Do not solve these integrals.*

   (a) The volume of rotation *about the x-axis* of the region bounded by $g(x) = e^{-x}$ and the lines $y = 0$, $x = 0$, and $x = 1$.

Use washers:

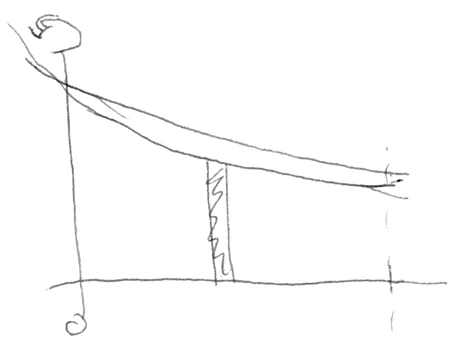


$$\int_0^1 \pi (e^{-x})^2 \, dx$$

$$= \boxed{\int_0^1 \pi e^{-2x} \, dx}$$

   (b) The volume of rotation *about the y-axis* of the region bounded by $g(x) = e^{-x}$, and the lines $y = 0$, $x = 0$, and $x = 1$.

Cylindrical shells:



$$\boxed{\int_0^1 2\pi x e^{-x} \, dx}$$

   (c) Which axis of rotation results in a larger volume? Justify your answer.

- Rotating About the $\boxed{\text{y-axis}}$ gives a larger volume.
- X-axis gives: $\int_0^1 \pi e^{-2x} dx = \frac{-\pi}{2} e^{-2x} \Big|_0^1 = \frac{\pi}{2}\left(1 - \frac{1}{e^2}\right) \approx 1.358$
- y-axis gives: $\int_0^1 2\pi x e^{-x} dx = -2\pi(xe^{-x} + e^{-x})\Big|_0^1 = 2\pi\left[1 - \frac{2}{e}\right] \approx 1.660$

$\Rightarrow$ Unfortunately, this problem is more subtle than initially anticipated. No fast way to compare these two integrals, although visual estimation from the graph is suggestive. Full credit for all reasonable answers.