Name: ______________________________

Directions: Show all work. Answers without work generally do not earn points. This test has 60 points.

1. **[2 parts, 6 points each]** Solve the following linear systems.

(a)
$$\begin{array}{rcrcl} 3x & + & y & = & 5 \\ x & - & 5y & = & 1 \end{array}$$

(b)
$$\begin{array}{rcrcrcl} x & + & 2y & - & z & = & 3 \\ 4x & - & 6y & + & 3z & = & 2 \\ 6x & - & 2y & + & z & = & 8 \end{array}$$

2. **[6 parts, 2 points each]** Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -2 & -1 \\ 5 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}$.

   If possible, compute the following. If not possible, write "undefined".

   (a) $B^T + C$

   (b) AC

   (c) CA

   (d) $A + I_2$

   (e) A^2

   (f) B^2

3. **[6 points]** Prove that if $A\mathbf{x} = \mathbf{b}$ has more than one solution, then it has infinitely many solutions. (Hint: if $\mathbf{x_1}$ and $\mathbf{x_2}$ distinct solutions, show that appropriate linear combinations of $\mathbf{x_1}$ and $\mathbf{x_2}$ yield infinitely many solutions.)

4. **[6 points]** Let A and B be matrices of appropriate sizes. One statement is true and the other is false. Select the true statement and prove it.

   - If the rth row of A is all zeros, then the rth row of AB is all zeros.
   - If the rth column of A is all zeros, then the rth column of AB is all zeros.

5. **[6 points]** Give an example of a lower-triangular (3×3)-matrix with as many non-zero entries as possible.

6. **[6 points]** Let $A = \begin{bmatrix} 4 & 2 & -3 & 6 \\ 1 & 0 & 2 & 0 \\ 7 & 1 & 3 & -4 \end{bmatrix}$. Find a matrix in reduced row echelon form that is row-equivalent to A.

7. [**6 points**] Let $A = \begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix}$. Find elementary matrices $E_1, \ldots, E_4$ such that $E_4E_3E_2E_1A = I_2$.

8. **[6 points]** Let $A = \begin{bmatrix} 2 & -1 & 8 \\ 1 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix}$. If it exists, find A^{-1}. Otherwise, write "does not exist".