Math374 **Test #3** **17 November 2011**

Name: ________________________________

This test has 60 points (10 points per page) but is scored out of 50 points. Scores are truncated at 50.

1. [**4 points**] A survey is conducted on television advertisements. A total of 15 commercials used music, 11 displayed text, and 12 used a narrator. Also, 2 commercials used a narrator and music, 5 used a narrator and text, and 3 used music and text. Finally, 2 commercials used music, text, and a narrator, and 1 commercial used none of these. In total, how many commercials are in the survey?

2. [**2 parts, 3 points each**] A trade school offers 50 classes. Every student takes 4 classes each semester.

   (a) How many students must the school have to guarantee that there are two students with exactly the same course schedule?

   (b) Suppose that the school has 1827 full time students. What can you say about the number of students registered for the largest class?

3. [**2 points**] State the mathematical relationship between $C(n,k)$ and $P(n,k)$.

4. [**2 points**] State the binomial theorem.

5. Find the following coefficients. Except in part (a), you may leave your answer in terms of permutation numbers (e.g. $P(n,r)$), binomial coefficients (e.g. $C(n,r)$), and factorials (e.g. $n!$).

   (a) [**2 points**] Find the numerical value of the coefficient of x^8 in $(x-2)^{12}$.

   (b) [**2 points**] Find the coefficient of x^3y^6 in $(2x+3y)^9$.

   (c) [**2 points**] Find the coefficient of x^4y in $(3x-y+2)^{14}$.

6. [**6 points**] Use the Euclidean algorithm to find $\gcd(1734, 1628)$ and express it as a linear combination of 1734 and 1628. Show your work.

7. [**4 points**] How many numbers in $\{1, 2, \ldots, 999\}$ are relatively prime to 1000?

8. [**4 points**] Give an example of a relation on $\{1, 2, 3\}$ that is reflexive, symmetric, and not transitive.

9. [**4 points**] Consider the equivalence relation ρ on $\mathbb{Z} \times \mathbb{Z}$ defined by $(x_1, y_1)\ \rho\ (x_2, y_2) \leftrightarrow x_1 y_1 = x_2 y_2$. Which ordered pairs in $\mathbb{Z} \times \mathbb{Z}$ are in the equivalence class of $(0, 0)$? Describe the equivalence class of $(0, 0)$.

10. [**4 parts, 3 points each**]

(a) Give an example of a function from $\{1, 2, 3\}$ to $\{a, b, c, d, e\}$ which is one-to-one/injective but not surjective/onto.

(b) How many one-to-one/injective functions are there from $\{1, 2, 3\}$ to $\{a, b, c, d, e\}$?

(c) Give an example of an onto/surjective function from $\{a, b, c, d, e\}$ to $\{1, 2, 3\}$.

(d) How many onto/surjective functions from $\{a, b, c, d, e\}$ to $\{1, 2, 3\}$ are there? Hint: count the complement. Let A_1 be the set of functions that map nothing to 1. Let A_2 be the set of functions that map nothing to 2. Let A_3 be the set of functions that map nothing to 3. What is $|A_1 \cup A_2 \cup A_3|$?

11. [**2 points**] A 6-slot database uses a hashing strategy to store numbers; the hash function is $h(x) = x \mod 6$. Initially, the database is empty. Show a picture of the hash table after the numbers $843, 145, 1932, 533, 204$ are inserted in the given order. Collisions are resolved by chaining.

12. [**2 parts, 4 points each**] In the RSA algorithm, let $p = 47$ and $q = 113$, so that $n = 5311$ and $\varphi(n) = 5152$. Pick $e = 13$.

    (a) Use the Euclidean algorithm to find d. Show your work.

    (b) Encode the plaintext message $T = 1024$. Show your work.