Name: ______________________________

This test has 6 pages, each worth 10 points. The test is scored out of 50 points. Your lowest scoring page is dropped.

1. [**2 points**] What conclusion(s), if any, can be reached from the following given hypotheses? If Sam does not rush, then Sam will be late. If Sam is late, then Sam will be fired. Sam is not late.

2. [**5 parts, 1 point each**] Using the given statement letters, translate the following sentences into wffs.

C: The cat looks guilty	A: The cat eats the bird	S: The bird sings
M: The owner is mad at the cat	P: The cat purrs	E: The bird escapes

   (a) The bird escapes or the cat eats the bird.

   (b) If the owner is mad at the cat, then the cat does not purr.

   (c) The cat looks guilty if and only if it eats the bird.

   (d) The owner is mad at the cat, and the bird sings only if it escapes.

   (e) The cat purrs unless the owner is mad at the cat.

3. [**3 parts, 1 point each**] Write the negation of the following statements using simple and natural English sentences.

   (a) The bird escapes or the cat eats the bird.

   (b) If the owner is mad at the cat, then the cat does not purr.

   (c) The cat looks guilty if and only if it eats the bird.

4. Two parts.

   (a) [**4 points**] Write a truth table for the following wff:

$$((A \to B) \leftrightarrow (B \to A))'$$

   (b) [**1 point**] Is the wff a tautology, a contradiction, or neither?

5. [**5 points**] Prove that the following wff is a tautology by giving a proof sequence.

Derivation Rule	Name/Abbreviation for Rule
$P \vee Q \iff Q \vee P$ $P \wedge Q \iff Q \wedge P$	Commutative—comm
$(P \vee Q) \vee R \iff P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \iff P \wedge (Q \wedge R)$	Associative—ass
$(P \vee Q)' \iff P' \wedge Q'$ $(P \wedge Q)' \iff P' \vee Q'$	De Morgan's laws—De Morgan
$P \to Q \iff P' \vee Q$	Implication—imp
$P \iff (P')'$	Double negation—dn
$P \leftrightarrow Q \iff (P \to Q) \wedge (Q \to P)$	Defn of Equivalence—equ
$\left.\begin{matrix} P \\ P \to Q \end{matrix}\right\} \implies Q$	Modus ponens—mp
$\left.\begin{matrix} P \to Q \\ Q' \end{matrix}\right\} \implies P'$	Modus tollens—mt
$\left.\begin{matrix} P \\ Q \end{matrix}\right\} \implies P \wedge Q$	Conjunction—con
$P \wedge Q \implies \left\{\begin{matrix} P \\ Q \end{matrix}\right.$	Simplification—sim
$P \implies P \vee Q$	Addition—add

$$((B \wedge C) \to A) \wedge C \wedge A' \to B'$$

Column 1:

Column 2:

6. [**6 parts, 1 point each**] Using the given predicates, translate the following sentences into wffs. The domain is all people in the world.

$O(x, y)$: x is at least as old as y	$M(x)$: x is married	$W(x)$: x is a woman
$F(x, y)$: x and y are friends	$R(x)$: x is rich	$S(x)$: x is a student

   (a) There is a rich student.

   (b) There are some rich people who are friends with everyone.

   (c) Every woman has a younger friend.

   (d) Some students are only friends with unmarried people.

   (e) Only women are friends with married students.

   (f) The oldest person in the world is not rich.

7. [**4 parts, 1 point each**] Write the negation of the following statements using simple and natural English sentences.

   (a) There are some rich people who are friends with everyone.

   (b) Every woman has a younger friend.

   (c) Some students are only friends with unmarried people.

   (d) Only women are friends with married students.

8. [**5 parts, 2 points each**] Determine whether the following sentences are valid. If the sentence is not valid, give an interpretation under which the sentence is false. Clearly indicate the domain of any interpretation you give.

   (a) $\forall x\,[Q(x)] \vee \forall x\,[Q(x)']$

   (b) $\forall x\,[P(x) \to Q(x)] \wedge \forall x\,[P(x)] \to \forall x\,[Q(x)]$

   (c) $(\forall x\,[P(x)] \to \forall x\,[Q(x)]) \to \forall x\,[P(x) \to Q(x)]$

   (d) $\forall x\,[P(x) \vee Q(x)] \wedge \exists x\,[Q(x)] \to \exists x\,[P(x)]$

   (e) $\forall x\,[\exists y\,[P(x,y)]] \to \forall y\,[\exists x\,[P(x,y)]]$

9. [**5 points**] Give a formal proof sequence that the following formula is valid.

$$\forall x \left[Q(x) \vee \forall y \left[P(x,y)\right]\right] \wedge \exists x \left[Q(x)'\right] \to \exists x \left[\forall y \left[P(x,y)\right]\right]$$

10. [**5 points**] Prove that the sum of two odd integers is even.

11. [**5 points**] Prove that the sum of two rationals is rational.

12. [**5 points**] Prove that an integer n is even if and only if n^2 is even.