Name: $\qquad$

1. [2 points] How many equivalence relations are there on $\{1,2,3\}$ ?
2. [2 points] A 6-slot database uses a hashing strategy to store numbers; the hash function is $h(x)=x \bmod 6$. Initially, the database is empty. Show a picture of the hash table after the numbers $843,145,1932,533,204$ are inserted in the given order. Collisions are resolved by linear probing.
3. [2 points] Let $A=\{1,2,3,4,5,6,7,8\}$. We consider permutations on $A$.
(a) Let $f=(25471) \circ(8742)$. Express $f$ as the composition of disjoint cycle permutations.
(b) Find the inverse $f^{-1}$ in tabular form.
4. [2 points] Decide whether the given functions are one-to-one/injective, onto/surjective, or bijective. For each blank cell in the table, write "Yes" if the function has the property, and "No" otherwise. You do not need to show your work.

In the following, let $A^{*}$ be the set of finite strings of $a$ 's and $b$ 's. For example, $a a b a, b b$, and the empty string $\lambda$ are all in $A^{*}$. Recall that $\mathbb{N}=\{0,1,2, \ldots\}$ and $\mathbb{Z}$ is the set of integers.

| Function | one-to-one | onto | bijective |
| :--- | :--- | :--- | :--- |
| $f: \mathbb{Z} \rightarrow \mathbb{Z}$ where $f(x)=x+6$ |  |  |  |
| $f: \mathbb{Z} \rightarrow \mathbb{Z}$ where $f(x)=x^{2}-1$ |  |  |  |
| $f: \mathbb{Z} \rightarrow \mathbb{Z}$ where $f(x)=x^{3}-1$ |  |  |  |
| $f: A^{*} \rightarrow \mathbb{N}$ where $f(x)$ equals the length of $x$ |  |  |  |
| $f: A^{*} \rightarrow A^{*}$ where $f(x)=x x$ |  |  |  |
| $f: A^{*} \rightarrow A^{*}$ where $f(x)$ equals the reverse of $x$ |  |  |  |

5. [2 points] In RSA, let $p=47$ and $q=43$. Then $n=2021$ and $\phi(n)=1932$. Pick $e=541$. Use the Euclidean algorithm to find the value of $d$.
