Math374 **Quiz #5** **3 November 2011**

Name: Solutions

1. [**2 points**] A job advertisement attracts 67 applicants. Of the applicants, a total of 29 have work experience, 16 have an advanced degree, 8 have corporate contacts, 7 have work experience and an advanced degree, 3 have an advanced degree and corporate contacts, 3 have work experience and corporate contacts, and 1 person has all three favorable attributes. How many applicants possess none of the three favorable attributes?

$A = \{x : x \text{ has work experience}\}$
$B = \{x : x \text{ has adv. degree}\}$
$C = \{x : x \text{ has contacts}\}$.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$
$$= 29 + 16 + 8 - 7 - 3 - 3 + 1$$
$$= 41$$

with no favorable attributes:

$$67 - 41 = \boxed{26}$$

2. [**2 points**] Find the exact numerical value of $\cancel{C(10,4)}$ $C(11,4)$ (also known as $\binom{\cancel{10}\,11}{4}$).

$$\cancel{\frac{10!}{6! \cdot 4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{210}} \qquad \frac{11!}{7! \, 4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{330}$$

3. [**2 points**] How many non-negative integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 50?$$

For example, there are four solutions where one of the variables is 50 and the rest are 0. You may leave your answer in terms of permutation numbers (e.g. $P(n,r)$), binomial coefficients (e.g. $C(n,r)$), and factorials (e.g. $n!$).

Stars/bars: 50 stars, 3 bars.

$$\boxed{C(53,3) \quad \text{or} \quad \binom{53}{3}}$$

4. **[2 points]** Find the coefficient of x^7 in $(7x-2)^{23}$. You may leave your answer in terms of permutation numbers (e.g. $P(n,r)$), binomial coefficients (e.g. $C(n,r)$), and factorials (e.g. $n!$).

$$(A+B)^n = \sum_{k=0}^{n} C(n,k)\, A^{n-k} B^k$$

$A = 7x$

$B = -2$

WANT: $A^7 B^{16}$, so chose $k=16$.

$C(23,16)\cdot(7x)^7\cdot(-2)^{16}$

$= \boxed{7^7\cdot(-2)^{16}\cdot C(23,16)}\, x^7$

5. **[2 points]** Give an example of a relation on $\{1,2,3\}$ that is reflexive, symmetric, and not transitive.

Try $\boxed{\rho = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}.}$

6. **[1 bonus point]** A 6×6-board with is tiled with 2×1 dominos. Prove that it is possible to divide the board in two pieces along a vertical or horizontal line without cutting any of the dominos.

only 18 dominos, a contradiction.

See me for more details. The basic idea is to give a proof by contradiction. There are 10 lines where we may cut; each crosses an even number of dominos. Also, each domino is cut by ~~only~~ exactly one line. If all lines cut dominos, then each line cuts at least 2, so ≥ 20 dominos are cut. But a 6×6 board is tiled with