Name: _____
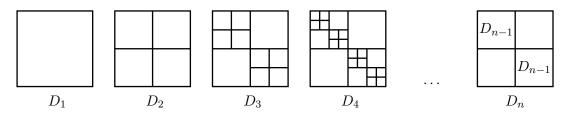
1. A sequence of geometric designs is defined recursively. The first design D_1 is a square. The nth design D_n is obtained by dividing a square into four quadrants. The upper left and lower right quadrants are scaled down copies of D_{n-1}.

D_1 D_2 D_3 D_4 ... D_n

(a) [**1 point**] The number of regions in D_1 is 1, in D_2 is 4, and in D_3 is 10. How many regions are in D_4, D_5, D_6, and D_7?

D_4: _____ D_5: _____ D_6: _____ D_7: _____

(b) [**1.5 points**] Define a recurrence relation $R(n)$ so that $R(n)$ is the number of regions in D_n.

(c) [**1 bonus point**] Solve the recurrence you obtained in part (b).

2. Let $A = \{\{3\}, 5, \{6\}, 8\}$, $B = \{\varnothing, \{3\}, 4, 5, 6\}$, and $C = \{\varnothing, \{4, 6\}\}$.

    (a) [**6 parts, 0.5 points each**] Which of the following statements are true?

          i. $\{4, 5, 6\} \subseteq B$ iv. $3 \in A$

          ii. $\{6\} \in A$ v. $\{4, 6\} \subseteq C$

          iii. $\varnothing \subseteq A$ vi. $\{4, 6\} \in C$

    (b) [**0.5 points**] Find $B \cap C$.

3. Let $T(n)$ be the following recurrence. $T(1) = 0$, $T(2) = 1$, and $T(n) = 10T(n-1) - 25T(n-2)$ for $n \geq 3$.

    (a) [**0.5 points**] Find the first five values of the sequence $T(n)$ from $T(1)$ to $T(5)$.

    (b) [**1.5 points**] Solve the recurrence.

4. [**2 parts, 1 point each**] An ATM pin number is a sequence of 4 digits.

   (a) How many pin numbers read the same forwards and backwards? For example, 2332 and 0000 count, but 9279 does not.

   (b) How many pin numbers contain at least one 7? For example, 7284 and 4727 count, but 1234 does not.