Name: Solutions

1. **[2.5 points]** Verify the correctness of the following program segment.

```
{x ≤ y}
if y > 10 then
   x = 4
else
   y = 12
end if
{x ≤ 10}
```

① Check: $\{x \le y \wedge y > 10\}\ x = 4\ \{x \le 10\}$: This is valid since $x = 4$ and $4 \le 10$.

② Check: $\{x \le y \wedge (y > 10)'\}\ y = 12\ \{x \le 10\}$: Since $(y > 10)'$ means that $y \le 10$ and $x \le y$ is a precondition, $x \le y \le 10$.

By ① and ② and the conditional inference rule, the code is valid.

2. **[2.5 points]** The loop inference rule of program correctness only ensures that the loop statement is "partially correct". Which crucial property of a working program is not guaranteed by the loop inference rule?

The program might not terminate/halt.

3. **[2.5 points]** Use Euclid's algorithm to compute gcd(15708, 1870). You may use a calculator for arithmetic, but show all other work involved in executing the algorithm.

$15708 = 8 \cdot 1870 + 748$ $\quad \gcd(15708, 1870) = \gcd(1870, 748)$

$1870 = 2 \cdot 748 + 374$ $\quad \gcd(1870, 748) = \gcd(748, 374)$

$748 = 2 \cdot 374 + 0$ $\quad \gcd(748, 374) = \gcd(374, 0)$

$= \boxed{374}$

4. **[2.5 points]** Use induction to prove that $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n\cdot(n+1)}=\frac{n}{n+1}$ for every positive integer n.

Proof: By induction on n.

Basis Step: When $n=1$, the LHS. is $\frac{1}{1\cdot 2}=\frac{1}{2}$ and the RHS. is $\frac{1}{1+1}=\frac{1}{2}$. Therefore the statement holds.

Inductive Step: Let $n\geq 2$. By the I.H.,

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)\cdot n}=\frac{n-1}{(n-1)+1}.$$

Adding $\frac{1}{n(n+1)}$ to both sides yields

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)\cdot n}+\frac{1}{n\cdot(n+1)}=\frac{n-1}{n}+\frac{1}{n(n+1)}.$$

The RHS of this equation simplifies:

$$\frac{n-1}{n}+\frac{1}{n(n+1)}=\frac{(n-1)(n+1)}{n(n+1)}+\frac{1}{n(n+1)}$$

$$=\frac{n^2-1}{n(n+1)}+\frac{1}{n(n+1)}=\frac{n^{\cancel{2}}}{\cancel{n}(n+1)}=\frac{n}{n+1}.$$

It follows that the RHS and LHS are equal. ■