Name: Solutions

1. [**2 points**] What conclusion(s), if any, can be reached from the following given hypotheses? If Sam does not rush, then Sam will be late. If Sam is late, then Sam will be fired. Sam is not late.

$\boxed{\text{Sam rushes.}}$

(Note: no conclusion is possible about whether Sam will be fired.)

2. [**2 points**] Using the given derivation rules, give a proof sequence for the following wff.

Derivation Rule	Name/Abbreviation for Rule
$P \vee Q \iff Q \vee P$ $P \wedge Q \iff Q \wedge P$	Commutative—comm
$(P \vee Q) \vee R \iff P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \iff P \wedge (Q \wedge R)$	Associative—ass
$(P \vee Q)' \iff P' \wedge Q'$ $(P \wedge Q)' \iff P' \vee Q'$	De Morgan
$P \to Q \iff P' \vee Q$	Implication—imp
$P \iff (P')'$	Double negation—dn
$P \leftrightarrow Q \iff (P \to Q) \wedge (Q \to P)$	Defn of Equivalence—equ
$\left.\begin{array}{r} P \\ P \to Q \end{array}\right\} \Longrightarrow Q$	Modus ponens—mp
$\left.\begin{array}{r} P \to Q \\ Q' \end{array}\right\} \Longrightarrow P'$	Modus tollens—mt
$\left.\begin{array}{r} P \\ Q \end{array}\right\} \Longrightarrow P \wedge Q$	Conjunction—con
$P \wedge Q \Longrightarrow \left\{\begin{array}{l} P \\ Q \end{array}\right.$	Simplification—sim
$P \Longrightarrow P \vee Q$	Addition—add

$$(B' \vee A) \wedge (B' \to C) \wedge C' \to A$$

1. C' hyp
2. $B' \to C$ hyp
3. $(B')'$ 1,2 ~~mp~~ mt
4. B 3, dn
5. $B' \vee A$ hyp
6. $B \to A$ 5, imp
7. A 4, mp.

3. **[2 parts, 1 point each]** Using the predicate symbols "$B(x)$: x is a ball", "$R(x)$: x is round", and "$S(x)$: x is a soccer ball", translate each sentence to a predicate wff. The domain is all objects in the world.

   (a) All soccer balls are round.

   $$\forall x\left[S(x) \to R(x)\right]$$

   (b) If all soccer balls are round, then some balls are not round.

   $$\forall x\left[S(x) \to R(x)\right] \to \exists x\left[B(x) \wedge R(x)'\right]$$

4. **[2 parts, 1 point each]** Give interpretations to prove that the following wffs are not valid.

   (a) $\exists x[A(x)] \wedge \exists x[B(x)] \to \exists x[A(x) \wedge B(x)]$.

   Domain: Integers

   $A(x)$: x is odd

   $B(x)$: x is even

   (b) $\big(\forall x[P(x) \to Q(x)]\big) \to \big(\exists x[P(x)] \to \forall x[Q(x)]\big)$.

   Domain: Integers

   ~~A(x)~~ $P(x)$: x is divisible by 4

   $Q(x)$: x is divisible by 2

5. [**2 points**] Prove that for all integers x and y, if xy is even, then x is even or y is even.

Proof: We show the contrapositive: if x is odd and y is odd, then xy is odd. Let x and y be odd integers. Because x and y are odd, $x = 2m+1$ and $y = 2n+1$ for some integers m and n. Therefore

$$\begin{aligned} xy &= (2m+1)(2n+1) \\ &= 4mn + 2n + 2m + 1 \\ &= 2(2mn + n + m) + 1. \end{aligned}$$

Because xy is one more than twice the integer $2mn + n + m$, it follows that xy is odd.

6. [**0 points**] Would you like to join a Math 374 study group? Please respond yes or no. By responding yes, you authorize me to include your name and USC email address in a message sent to your Math 374 study group. Each group will have at most 4 students chosen at random from the class.