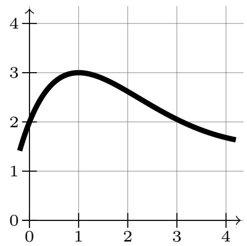
Name: ______________________________

Directions: Show all work. No credit for answers without work.

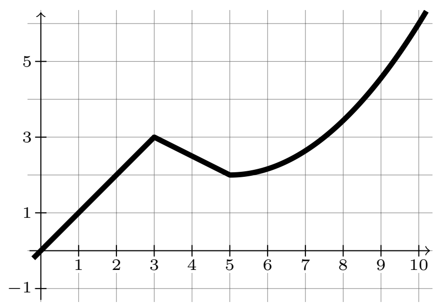
1. [**2 parts, 4 points each**] The graph of $f(x)$ appears below.

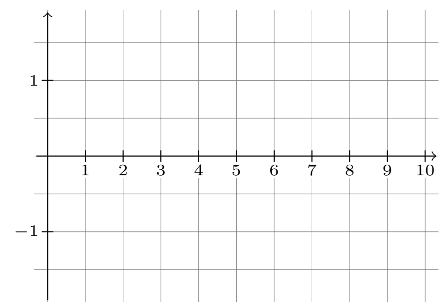


(a) Sketch the tangent line to $f(x)$ at $x = 3$ in the provided graph.

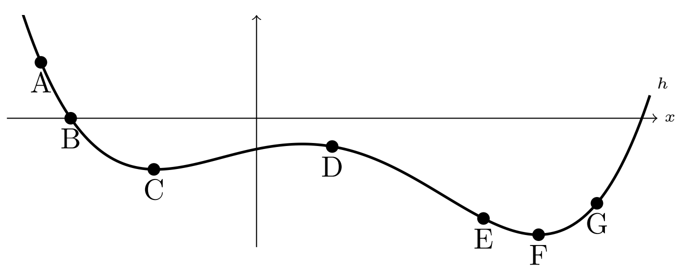
(b) Estimate $f'(3)$.

2. [**6 points**] The graph of $g(x)$ appears below. Sketch $g'(x)$ in the space provided.





3. [**2 parts, 3 points each**] The following is a graph of $h(x)$. Some points are labeled.



(a) At which of the labeled points is the <u>second derivative</u> $h''(x)$ positive?

(b) At which of the labeled points is the <u>second derivative</u> $h''(x)$ negative?

4. [**4 parts, 2 points each**] A glass of water is removed from the refrigerator and placed on the counter. The temperature T of the water (in degrees Fahrenheit) is a function $T = f(x)$ of the time x (in minutes) since the water is exposed to room temperature.

   (a) In $f(15) = A$, what are the units of 15? What are the units of A?

   (b) Do you expect the derivative f' to be positive or negative?

   (c) In the statement $f'(15) = B$, what are the units of B?

   (d) Do you expect the second derivative f'' to be positive or negative?

5. [**2 parts, 3 points each**] Fill in the blanks. If $f''(x) > 0$, then

   (a) $f'(x)$ is ____, and

   (b) $f(x)$ is ____.

6. [**3 parts, 2 points each**] Let $C(q)$ be the cost (in dollars) of producing q items, and let $R(q)$ be the revenue (in dollars) received when producing q items.

   (a) If $C(40) = 2320$ and $C'(40) = 15$, estimate $C(43)$.

   (b) If $C'(40) = 15$ and $R'(40) = 18$, estimate the profit that results from producing the 41st item.

   (c) The current production level is 67 items, and $C(67) = 4208$, $C'(67) = 24$, $R(67) = 3100$, and $R'(67) = 32$. In these circumstances, should the company increase production or decrease production? Why?

7. [**10 parts, 2 points each**] Differentiate the following functions.

(a) $y = 2x^8$

(b) $y = \dfrac{4}{x^5}$

(c) $y = \sqrt{x}$

(d) $y = 3x^7 - x^2$

(e) $y = e^{-x}$

(f) $y = 4^x$

(g) $y = e^x + x^e$

(h) $y = \ln(x)$

(i) $y = 2(1.09)^x + x^{1.2} + \ln\left(\sqrt{5}\right)$

(j) $y = \dfrac{e^3 - \ln(\ln(4.26))}{2\pi + \sqrt{11}\ln(3)}$

8. [**4 parts, 5 points each**] Differentiate the following functions.

   (a) $y = (x^2 + 6x + 1)^{15}$

   (b) $y = \dfrac{x^3 - x^2}{e^x + 5}$

   (c) $y = x^2 e^{7x}$

   (d) $y = \ln\left(x \ln(x)\right)$

9. [**8 points**] Find the equation of the line tangent to the function $f(x) = (2x - 1)^3$ at $x = 2$.

10. Let $g(x) = (x - 4)^3(2x + 1)^2$.

    (a) [**6 points**] Find $g'(x)$ in factored form.

    (b) [**6 points**] Make a sign chart for $g'(x)$ and classify each critical point of $g(x)$ as a local minimum, a local maximum, or neither.