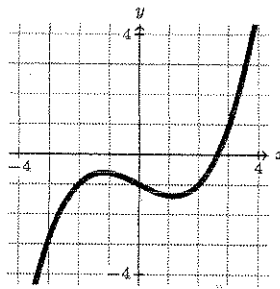Name: _____ *Key* _____

Multiple Choice

Directions: Read all questions carefully. In this section, you do not need to show your work. Mark the box that corresponds to the *best* answer. Unless otherwise directed, mark one box only. If you would like to change your answer, completely erase your old answer.

(3$^{\text{pts}}$)   **1.** The amount of water W (in thousands of gallons) in a pool is a function of time t (in days) since it was filled. Translate the statement $W(15) = 12$ into English.

☐ For every 15 days, the pool looses 12 gallons of water.

☐ When the pool has 15,000 gallons of water, it has been 12 days since it was filled.

☐ For every 12 days, the pool gains 15 gallons of water.

☐ When the pool has 12,000 gallons of water, it has been 15 days since it was filled.

☐ After 12 days, the pool contains 15 thousand gallons of water.

☒ After 15 days, the pool contains 12 thousand gallons of water.

($2^{\text{pts}}_{\text{ea.}}$)   **2.** Graphs of $f(x)$ and $g(x)$ appear below.
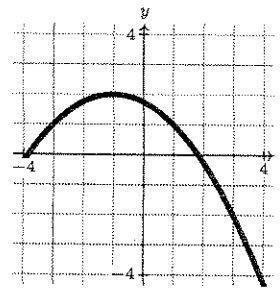
$f(x)$ $g(x)$

(a) Find $g(-3)$.

☐ -2   ☐ 3.5   ☐ 4   ☐ -4   ☐ -3
☐ 3   ☐ -1   ☐ 0   ☒ 1   ☐ 2

(b) Find $f(g(3))$.

☐ -4   ☐ 1   ☒ -1   ☐ 3   ☐ -3
☐ 2   ☐ 4   ☐ -2   ☐ -0.5   ☐ 0

(c) Find $g(f(3))$.

☐ -2   ☐ 2   ☐ 3   ☐ 0   ☐ -1
☐ -3   ☒ 1   ☐ -0.5   ☐ 4   ☐ -4

(d) Find the average rate of change in $f(x)$ between $x = -2$ and $x = 3$.

☐ -0.4   ☐ 1   ☐ 2.5   ☐ 0   ☐ 2
☒ 0.4   ☐ -1.6   ☐ 1.6   ☐ -2.5   ☐ -1

(e) Find all value(s) of x such that $f(x) = -1$. *Mark all that apply.*

☒ 2   ☐ -0.5   ☐ -3   ☐ 3   ☐ -4
☒ -2   ☐ -1   ☒ 0   ☐ 4   ☐ 1

(5^{pts})   **3.** At the end of summer, a tree has 4,600 leaves. After 2 weeks, the tree has 4,100 leaves. Find the relative rate of change in the number of leaves.

☐ -250 leaves per week   ☐ -12.19%   ☐ 250 leaves per week

☐ 12.19%   ■ -10.87%   ☐ 10.87%

(2$^{pts}_{ea.}$)   **4.** Decide whether the following tables might represent linear functions, exponential functions, or neither.

x	$f(x)$		x	$g(x)$		x	$h(x)$		x	$r(x)$		x	$s(x)$
0	3		0	83		0	1.4582		0	3		0	12
1	5		1	66		1	1.1666		1	9		1	12
2	10		2	49		2	0.9332		2	27		2	12
3	9		3	32		3	0.7466		3	81		3	12
4	14		4	15		4	0.5973		4	243		4	12

also credit given if this is not marked

(a) Which table(s) might represent linear functions? *Mark all that apply.*

☐ f(x)   ■ s(x)   ■ g(x)   ☐ h(x)   ☐ r(x)

(b) Which table(s) might represent exponential functions? *Mark all that apply.*

☐ g(x)   ⟨■ s(x)⟩   ☐ f(x)   ■ h(x)   ■ r(x)

(c) Which table(s) represent neither? *Mark all that apply.*

☐ s(x)   ☐ g(x)   ☐ r(x)   ☐ h(x)   ■ f(x)

(5^{pts})   **5.** A radioactive substance with a half-life of 18 days is accidentally spilled in a laboratory. Safety regulations forbid anyone from entering the laboratory until at most 2.5% of the original amount spilled remains. How many days must pass before people be able to return to the laboratory?

☐ 133.6 days   ☐ 129.2 days   ☐ 720.0 days   ■ 95.8 days   ☐ 101.59 days

☐ Infinitely many ☐ 59.8 days   ☐ 56.4 days   ☐ 18.0 days   ☐ None of these

(3$^{pts}_{ea.}$)   **6.** You own a small business that has just negotiated a new contract. The contract calls for your company to receive an immediate payment of \$40,000, a payment of \$45,000 after 1 year, a payment of \$50,000 after 2 years, and a payment of \$55,000 after 3 years. Assume that invested cash earns interest at a rate of 3.2%, compounded continuously.

See last page for work

(a) Find the future value (in 3 years time) of the 4 payments made to your company.

☐ \$190,280   ☐ \$198,970   ☐ \$199,020   ☐ \$190,610   ☐ \$190,000

☐ \$198,490   ☐ \$190,390   ■ \$198,630   ☐ \$0   ☐ None of these

(b) Find the present value of these payments.

☐ \$0   ☐ \$181,448   ☐ \$190,000   ☐ \$190,391   ☐ \$181,212

☐ \$40,000   ■ \$180,449   ☐ \$180,876   ☐ \$180,593   ☐ None of these

(5^{pts})   **7.** Which discrete interest rate is equivalent to a continuous interest rate of 7.41%?

☐ 7.58%   ☐ 7.63%   ☐ 7.46%   ■ 7.69%

☐ 7.24%   ☐ 7.41%   ☐ 7.15%   ☐ 7.55%

$\left(3^{\text{pts}}_{\text{ea.}}\right)$ **8.** The following is a graph of the function $f(x)$. Some points are labeled.
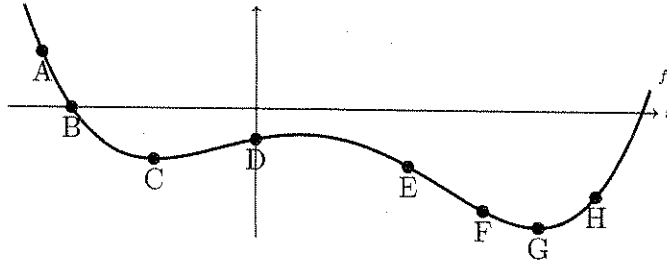

(a) At which of the labeled points is the derivative of f zero? *Mark all that apply.*

☐ A   ☐ F   ☐ H   ☐ B   ☐ E   ▣ G   ☐ D   ▣ C

(b) At which of the labeled points is the derivative of f negative? *Mark all that apply.*

☐ D   ▣ B   ▣ F   ☐ G   ▣ E   ▣ A   ☐ C   ☐ H

(c) At which of the labeled points is the derivative of f positive? *Mark all that apply.*

▣ D   ☐ A   ☐ B   ☐ C   ☐ E   ☐ G   ▣ H   ☐ F

(d) At which of the labeled points is the derivative of f most negative?

☐ B   ☐ H   ☐ C   ☐ E   ☐ F   ☐ D   ☐ G   ▣ A

Free Response

Directions: Show all work. No credit for answers without work.

$\left(2^{\text{pts}}_{\text{ea.}}\right)$ **9.** Sketch graphs of functions with the following properties.
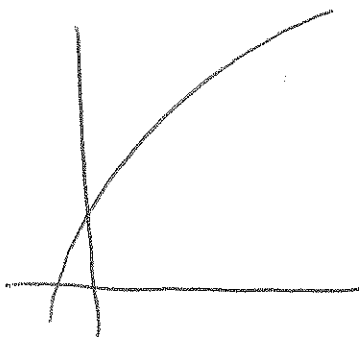
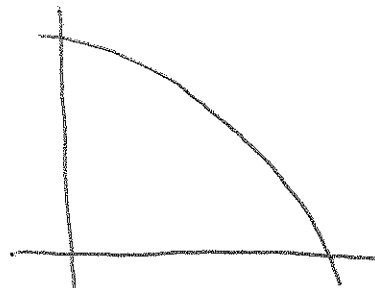(a) Increasing and concave up.
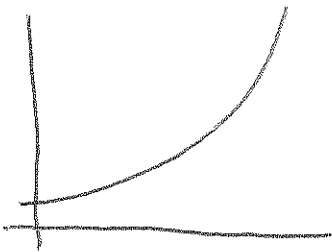
(c) Decreasing and concave up.
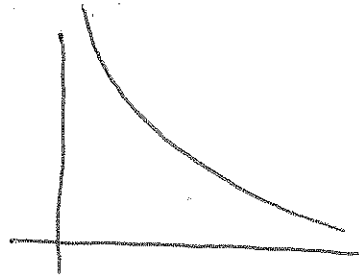
(b) Increasing and concave down.

(d) Decreasing and concave down.

$(3^{pts}_{ea.})$ **9.** Find formulas for the following functions.

    (a) The linear function through $(-2, 6)$ and $(3, 1)$.

$$m = \frac{Y_1 - Y_0}{X_1 - X_0} = \frac{1 - 6}{3 - (-2)} = \frac{-5}{5} = -1$$

8 pts

$$y = -1 \cdot x + b$$
$$6 = -1 \cdot (-2) + b$$
$$6 = 2 + b$$
$$b = 4$$

$$\boxed{y = -x + 4}$$

    (b) The exponential function through $(-2, 6)$ and $(3, 1)$.

6 pts

• $P = P_0 a^t$ 1 pt.

• $6 = P_0 a^{-2}$

• $1 = P_0 a^3$

Divide:

$$\frac{6}{1} = \frac{P_0 a^{-2}}{P_0 a^3}$$

$$6 = \frac{a^{-2}}{a^3}$$

$$6 = \frac{1}{a^3 \cdot a^2}$$

$$6 = \frac{1}{a^5}$$

$$a^5 = \frac{1}{6}$$

$$a = \left(\frac{1}{6}\right)^{\frac{1}{5}} = 6^{-\frac{1}{5}}$$

$$1 = P_0 \left(6^{-\frac{1}{5}}\right)^3$$

$$1 = P_0 \left(6^{\frac{1}{5}}\right)$$

$$1 = P_0 (6) 6^{-\frac{3}{5}}$$

$$P_0 = \frac{1}{(6^{-\frac{3}{5}})} = 6^{+\frac{3}{5}}$$

$$\boxed{P = 6^{\frac{3}{5}} \cdot 6^{-\frac{t}{5}}}$$

$$\approx \boxed{(2.93) \cdot (0.699)^t}$$

$(3^{pts}_{ea.})$ **10.** The quantity q (in millions of boxes) of corn flake cereal demanded by the market when the price of a box is p dollars is given by the equation $q = 44 - 5p$.

    (a) Find the p-intercept and q-intercept and interpret them in terms of consumer demand.

more space

p-intercept: $q = 0$.

$$0 = 44 - 5p$$
$$5p = 44$$

2: values// 1: interp.   q

$p = \boxed{8.8}$

Pts No work!

When the price is $\boxed{\$8.80}$ no one wants to buy.
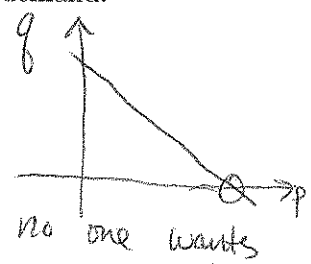
q-intercept: $p = 0$:
$$q = 44 - 5 \cdot 0 = \boxed{44}$$

• Even when the cereal is free, people only want 44 million cereal boxes.

(b) The supply curve is given by $q = p^2 + 2p$, where q (in millions of boxes) is the quantity of corn flakes produced when the price of a box is p dollars. Find the equilibrium price and quantity sold.

6.

Supply = demand

$p^2 + 2p = 44 - 5p$

$p^2 + 7p - 44 = 0$

$(p + 11)(p - 4) = 0$

$p = -11$ or $p = 4$

• Equilibrium price: $\boxed{\$4.00}$

• Equilibrium quantity:

$q = 4^2 + 2(4) = 16 + 8 = \boxed{24 \text{ million boxes}}$

$(3^{\text{pts}}_{\text{ea.}})$ 12. Solve the following equations for x <u>exactly</u>. Decimal approximations are worth partial credit.

(a) $3e^{2x} = 4$

9pts

4pts ea.

$e^{2x} = \dfrac{4}{3}$

$\ln(e^{2x}) = \ln\left(\dfrac{4}{3}\right)$

$2x = \ln(4) - \ln(3)$

$x = \boxed{\dfrac{\ln(4) - \ln(3)}{2}}$

(b) $5e^{6x+1} = 2^{2x}$

② for splitting.
$\ln(5 \cdot e^{6x+1})$

$\ln(5e^{6x+1}) = \ln(2^{2x})$

$\ln(5) + \ln(e^{6x+1}) = 2x \ln(2)$

$\ln(5) + 6x + 1 = 2x \ln(2)$

$6x - 2x \ln(2) = -1 - \ln(5)$

$x(6 - 2\ln(2)) = -1 - \ln(5)$

$x = \boxed{\dfrac{-1 - \ln(5)}{6 - 2\ln(2)}}$

(c) $4\ln(7x + 6) = 12$

$\ln(7x + 6) = 3$

$e^{\ln(7x+6)} = e^3$

$7x + 6 = e^3$

$7x = e^3 - 6$

$x = \boxed{\dfrac{e^3 - 6}{7}}$

(d) $8e^{-x+2} = \ln(3)$

$\ln(8e^{-x+2}) = \boxed{\ln(\ln(3))}$ ②

$\ln(8) + \ln(e^{-x+2}) = \ln(\ln(3))$

$\ln(8) \cdot -x + 2 = \ln(\ln(3))$

$\boxed{\ln(8) - \ln(\ln(3)) + 2} = x$ ✗

#5 $P = P_0 a^t$

$\frac{1}{2} = a^{18}$

$a = \left(\frac{1}{2}\right)^{\frac{1}{18}}$

$0.025 = \left(\left(\frac{1}{2}\right)^{\frac{1}{18}}\right)^t$

$\ln(0.025) = \frac{t}{18} \ln\left(\frac{1}{2}\right)$

$t = \dfrac{18 \ln(0.025)}{\ln\left(\frac{1}{2}\right)} = 95.79 \text{ days.}$

#6 $P = 40,000\, e^{0.032 \cdot 3} + 45000\, e^{0.032 \cdot 2} + 50000\, e^{0.032 \cdot 1} + 55000$

$44,030.36 + 47,974.16 + 51,625.88 + 55000$

$= \$198,630.40$

$P_t \;\; 198,630.4 = P_0\, e^{0.032(3)}$

$P_0 = \dfrac{198630.4}{e^{0.032(3)}} = \$180,448.6$

#7. $P = P_0\, e^{0.0741 t}$

$= P_0 \left(e^{0.0741}\right)^t$

$= P_0 (1.0769)^t$

     $P = P_0 (1 + 0.0769)^t$

$r = 0.0769 = \boxed{7.69\%}$