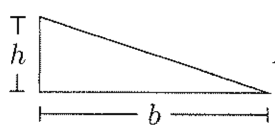
Name: Key

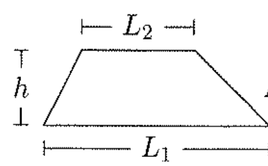
Do not turn the page until instructed.

Directions:

1. Write your name on this page.
2. Round all numerical answers to three (3) decimal places.
3. Show your work unless you are instructed otherwise. No credit for answers without work.
4. You may use a calculator provided it is not equipped with a Computer Algebra System (CAS).
5. Turn off and put away all other electronic equipment (especially cell phones), notes, and books.
6. Good luck!



Area $= \frac{1}{2}bh$



Area $= \frac{h}{2}(L_1 + L_2)$

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Section:	Test 1	Test 2	Test 3	Avg. Value
Max Points:	50	50	50	10
Points Earned:				

Begin Section: Test 1 Material

1. [**6 parts, 3 points each**] Short Answer Questions.

(a) Let $f(x) = (1-x)^3$ and $g(x) = \frac{x}{x+1}$. Find $g(f(-1))$.

$$g(f(-1)) = g((1-(-1))^3) = g(2^3)$$
$$= g(8) = \boxed{\frac{8}{9}}$$

(b) Find the equation of the line through $(2,1)$ and $(-3,2)$.

$$y - y_0 = m(x - x_0) \qquad \cdot\ m = \frac{1-2}{2-(-3)} = -\frac{1}{5}$$
$$y - 1 = -\frac{1}{5}(x-2)$$
$$\boxed{y = -\frac{1}{5}x + \frac{7}{5}}$$

(c) Solve for x in the equation $4\ln(2x+1) = 2$.

$$\ln(2x+1) = \frac{2}{4}$$
$$e^{\ln(2x+1)} = e^{1/2}$$
$$2x+1 = e^{1/2}$$
$$2x = e^{1/2} - 1$$
$$\boxed{x = \frac{e^{1/2}-1}{2}}$$

(d) Solve for x in the equation $e^{4-x} = 3$.

$$\ln(e^{4-x}) = \ln(3)$$
$$4 - x = \ln(3)$$
$$4 - \ln(3) = x$$
$$\boxed{x = 4 - \ln(3)}$$

(e) Solve for x in the equation $2^x = 3^{x+1}$.

$$\ln(2^x) = \ln(3^{x+1})$$
$$x\ln(2) = (x+1)\ln(3)$$
$$x\ln(2) = x\ln(3) + \ln(3)$$
$$x\ln(2) - x\ln(3) = \ln(3)$$
$$x(\ln(2) - \ln(3)) = \ln(3)$$
$$\boxed{x = \frac{\ln(3)}{\ln(2) - \ln(3)}}$$

(f) Find the average rate of change of the function $f(t) = t(t+1)$ over the interval $[-2, 3]$.

$$AROC = \frac{f(3)-f(-2)}{3-(-2)} = \frac{3\cdot 4 - (-2)\cdot(-1)}{5} = \frac{12-2}{5}$$

$$= \boxed{\frac{10}{5}} = \boxed{2}$$

2. [**3 parts, 5 points each**] At time $t = 0$ years, Town A has a population of 12.2 million and Town B has a population of 3.62 million.

(a) Experts project that the population of Town A will decline exponentially at a continuous annual rate of 2.7% for the foreseeable future. Give a formula for the population of Town A at time t.

$$P = P_0 e^{kt}$$

$$P = 12.2\, e^{-0.027t}$$

(b) Experts predict that at time $t = 1$ year, the population of Town B will be 3.71 million. Assuming that the population of Town B grows exponentially, give a formula for the population of Town B at time t.

~~3.62=~~

$$3.71 = 3.62\, e^{k\cdot 1}$$

$$1.025 \approx e^k$$

$$\ln(1.025) = k, \quad k \approx 0.0246$$

$$P = 3.62\, e^{0.0246t}$$

(c) When will the towns have the same population?

$$12.2\, e^{-0.027t} = 3.62\, e^{0.0246t}$$

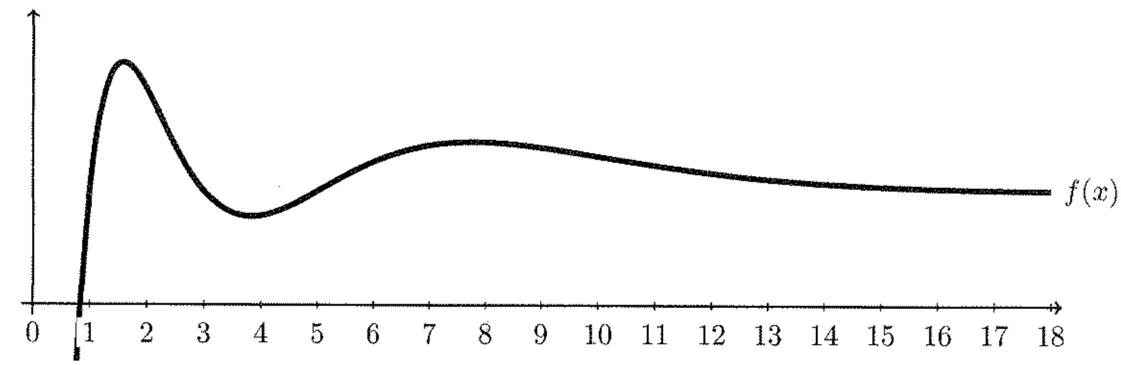
$$3.37\, e^{-0.027t} = e^{0.0246t}$$

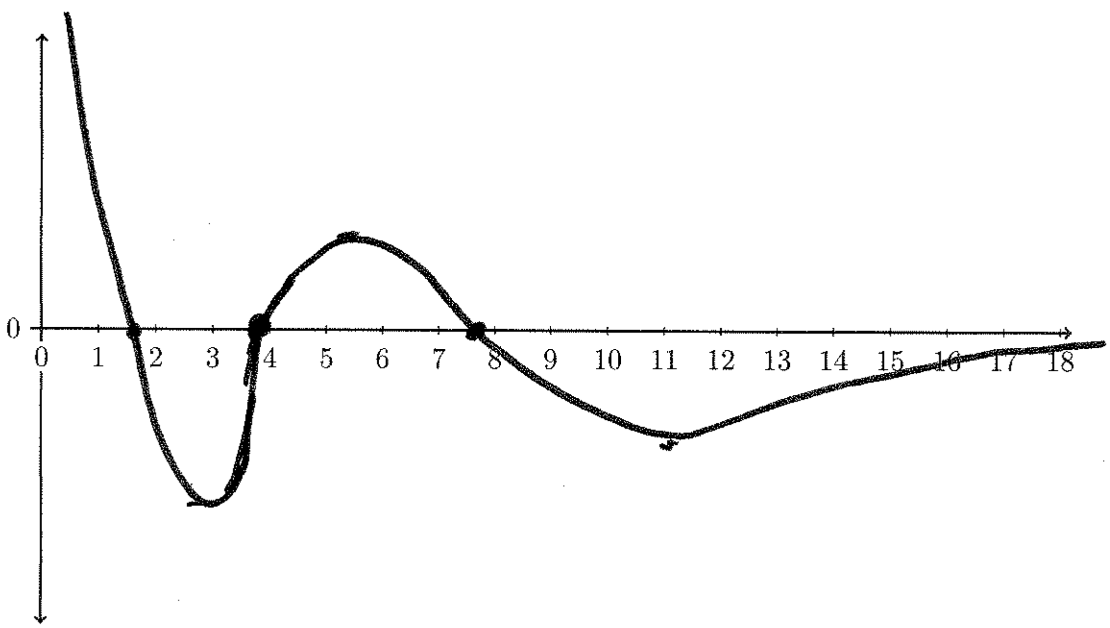
$$\ln(3.37) - 0.027t = 0.0246\, t$$

$$\ln(3.37) = 0.0516\, t$$

$$t \approx \boxed{23.54 \text{ years}}$$

3. [**10 points**] The graph of a function $f(x)$ appears below. In the space provided, sketch the derivative $f'(x)$. Your sketch should capture the important features of $f'(x)$, such as where $f'(x) = 0$, the local extrema of $f'(x)$, and the behavior of $f'(x)$ as x grows.





4. [**7 points**] Carbon-14 is a radioactive substance with a half-life of 5,730 years. A fossil is discovered that contains only 14% of its original Carbon-14. How old is the fossil?

$P = P_0 e^{kt}$

$\frac{1}{2} = 1 e^{k \cdot 5730}$

$\ln(\frac{1}{2}) = k \cdot 5730$

$k = \frac{\ln(\frac{1}{2})}{5730} \approx -0.000121$

$0.14 = 1 e^{-0.000121 t}$

$\ln(0.14) = -0.000121 t$

$t = \frac{\ln(0.14)}{-0.000121}$

$\approx \boxed{16{,}249 \text{ years}}$

Begin Section: Test 2 Material

1. [**10 parts, 2 points each**] Differentiate the following functions.

(a) $f(x) = 4$

$f'(x) = 0$

(b) $f(x) = -3x^3 + 8x^2 + 1$

$f'(x) = -9x^2 + 16x$

(c) $f(x) = \frac{1}{x}$ $= x^{-1}$

$f'(x) = -x^{-2}$

$= -\frac{1}{x^2}$

(d) $f(x) = 3^x$

$f'(x) = \ln(3) \cdot 3^x$

(e) $f(x) = e^{-3x}$

$f'(x) = -3 \cdot e^{-3x}$

(f) $f(x) = 4x^{1.5}$

$f'(x) = 4 \cdot 1.5\, x^{1.5-1}$

$= 6\, x^{0.5} = \boxed{6\sqrt{x}}$

(g) $f(x) = 2\ln(x)$

$f'(x) = \frac{2}{x}$

(h) $f(x) = e^{-x} + e^x$

$f'(x) = -e^{-x} + e^x$

(i) $f(x) = \frac{2+e^{-\sqrt{13}}}{\ln(8)+1}$

const!

$f'(x) = 0$

(j) $f(x) = x^{\ln(3)+1}$

$f'(x) = (\ln(3)+1)\, x^{\ln(3)}$

2. [**4 parts, 4 points each**] Differentiate the following functions.

(a) $f(x) = (x^2 + 5x)^7$

$$f'(x) = 7(x^2+5x)^6 \cdot \frac{d}{dx}\left[x^2+5x\right]$$

$$= \boxed{7(x^2+5x)^6(2x+5)}$$

(b) $f(x) = \ln(e^{\sqrt{x}} + 1)$

$$f'(x) = \frac{1}{e^{\sqrt{x}}+1} \cdot \frac{d}{dx}\left[e^{\sqrt{x}}+1\right]$$

$$= \frac{1}{e^{\sqrt{x}}+1} \cdot e^{\sqrt{x}} \cdot \frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{e^{\sqrt{x}}+1} \cdot e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \boxed{\frac{e^{\sqrt{x}}}{2\sqrt{x}(e^{\sqrt{x}}+1)}}$$

(c) $f(x) = \dfrac{x^2}{e^x + \ln x}$

$$f'(x) = \frac{(e^x+\ln x)\cdot 2x - x^2(e^x+\frac{1}{x})}{(e^x+\ln(x))^2}.$$

(d) $f(x) = \left(\dfrac{1+x}{1+x^2}\right)^5$

$$f'(x) = 5\left(\frac{1+x}{1+x^2}\right)^4 \cdot \frac{d}{dx}\left[\frac{1+x}{1+x^2}\right]$$

$$= \boxed{5\left(\frac{1+x}{1+x^2}\right)^4 \cdot \frac{(1+x^2)\cdot 1 - (1+x)\cdot 2x}{(1+x^2)^2}}$$

3. [**4 points**] When a toy company produces 19 toys, the total cost is \$4221 and the marginal cost is \$15 per toy. Estimate the total cost of producing 17 toys.

$$\Delta C = MC \cdot \Delta q = 15(-2) = -30$$

$$C(17) \approx C(19) + \Delta C = 4221 - 30 = \boxed{\$4191}$$

4. [**10 points**] Find and classify the critical points of $f(x) = (2x+1)^2 e^x$ as local minima, local maxima, or neither.

$$f'(x) = 2(2x+1) \cdot 2 \cdot e^x + (2x+1)^2 \cdot e^x$$

$$= (2x+1)e^x \left[4 + (2x+1)\right]$$

$$= (2x+1)e^x(2x+5)$$

Crit pts: $(2x+1)(2x+5)e^x = 0$

$2x+1=0$ or $2x+5=0$ or $e^x = 0$

$x = -\frac{1}{2}$ $\quad x = -\frac{5}{2}$ $\quad$ No soln

FDT:

f: $\nearrow$ $\searrow$ $\nearrow$

sign f': + 0 − 0 +

$-5/2$ $\quad -\frac{1}{2}$

local max $\quad$ local min

Begin Section: Test 3 Material

1. [**2 parts, 9 points each**] Find the points of inflection of the following functions.

(a) $f(x) = \frac{1}{12}x^4 - x^3 + \frac{9}{2}x^2$

$$f'(x) = \frac{1}{3}x^3 - 3x^2 + 9x$$

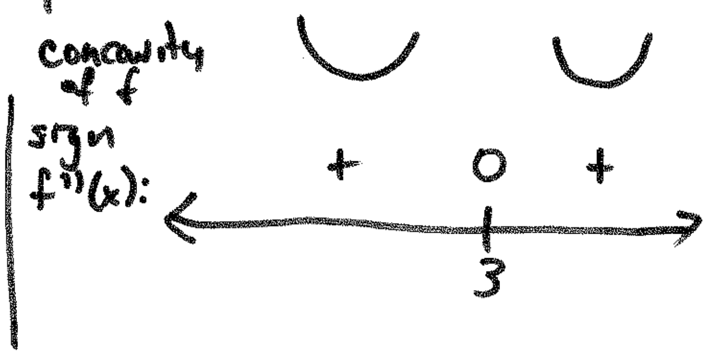
$$f''(x) = x^2 - 6x + 9$$

Make sign chart for $f''(x)$:

$$x^2 - 6x + 9 = 0$$

$$(x-3)(x-3) = 0$$

$$x = 3$$



No change in concavity, so f has no inflection points.

(b) $g(x) = \frac{1}{12}x^4 - \frac{1}{2}x^2$

$$g'(x) = \frac{1}{3}x^3 - x$$

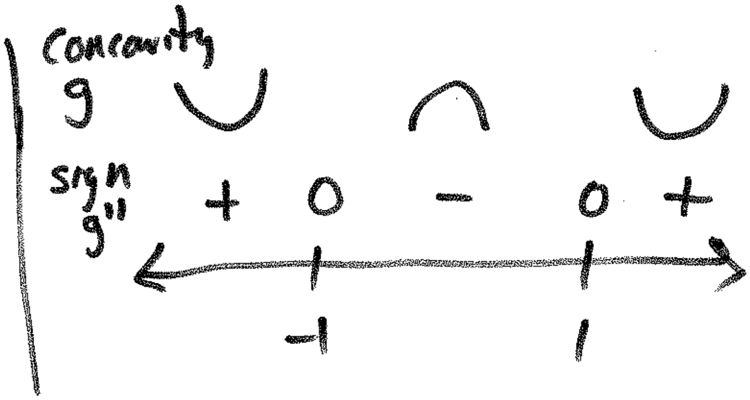
$$g''(x) = x^2 - 1$$

Sign chart for $g''(x)$:

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$x = 1 \text{ or } x = -1$$



Since the concavity of g changes at $\boxed{-1}$ and $\boxed{+1}$, these are inflection points of g.

2. [**8 parts, 2 points each**] Evaluate the following indefinite integrals.

(a) $\int 5\,dx$

$5x + C$

(b) $\int z - 3z^2\,dz$

$\frac{z^2}{2} - z^3 + C$

(c) $\int e^{0.5t}\,dt$

$\frac{1}{0.5} e^{0.5t} + C$

$= \boxed{2e^{0.5t} + C}$

(d) $\int x^3(x-1)\,dx$

$= \int (x^4 - x^3)\,dx$

$= \boxed{\frac{x^5}{5} - \frac{x^4}{4} + C}$

(e) $\int \frac{2}{x}\,dx$

$= 2\ln|x| + C$

(f) $\int r^{2.4}\,dr$

$= \frac{r^{3.4}}{3.4} + C$

(g) $\int \ln(3)\,dx$ const ~~thee~~ function

$= \ln(3)\cdot x + C$

(h) $\int x^{e-1}\,dx$ power rule

$= \boxed{\frac{x^e}{e} + C}$

3. [**4 parts, 4 points each**] Solve the following definite integrals exactly. Your answers may involve logarithmic and/or exponential functions. Show your work.

(a) $\int_1^2 x^2\,dx$

$= \frac{x^3}{3}\Big|_1^2$

$= \frac{2^3}{3} - \frac{1^3}{3}$

$= \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}$

(b) $\int_1^8 \frac{1}{\sqrt{3t+1}}\,dt$

$w = 3t+1$

$\frac{dw}{dt} = 3$

$\frac{1}{3}dw = dt$

$\int \frac{1}{\sqrt{w}} \cdot dt$

$= \int \frac{1}{\sqrt{w}} \cdot \frac{1}{3}dw$

$= \frac{1}{3}\int w^{-1/2}\,dw$

$= \frac{1}{3}\left(2w^{1/2}\right) + c$

$= \left(\frac{2}{3}\sqrt{3t+1}\right)\Big|_1^8$

$= \frac{2}{3}\left(\sqrt{24+1}\right) - \frac{2}{3}\sqrt{3+1}$

$= \frac{2}{3}\cdot 5 - \frac{2}{3}\cdot 2 = \boxed{2}$

(c) $\int_0^1 (x+1)(x^2+2x)^3\,dx$

$w = x^2+2x$

$\frac{dw}{dx} = 2x+2$

$\frac{1}{(2x+2)}\cdot dw = dx$

$\int (x+1)\,w^3 \frac{1}{2x+2}$

$= \int \frac{1}{2} w^3\,dw$

$= \frac{1}{2}\frac{w^4}{4} + C$

$= \frac{(x^2+2x)^4}{8}\Big|_0^1$

$= \frac{(1^2+2)^4}{8} - \frac{0^4}{8} = \boxed{\frac{81}{8}}$

(d) $\int_1^{e^2} \frac{(\ln x)^3}{x}\,dx$

$w = \ln x$

$\frac{dw}{dx} = \frac{1}{x}$

$x\cdot dw = dx$

$\int \frac{w^3}{\cancel{x}} \cdot \cancel{x} \cdot dw$

$= \frac{w^4}{4} + c$

$= \frac{(\ln(x))^4}{4}\Big|_1^{e^2}$

$= \frac{(\ln(e^2))^4}{4} - \frac{(\ln(1))^4}{4}$

$= \frac{2^4}{4} - \frac{0^4}{4} = \boxed{4}$

Begin Section: Average Value

1. [**5 points**] Find the average value of the function $f(x) = 1/x^2$ over the interval $[1, 3]$.

$$AV = \frac{1}{3-1}\int_1^3 \frac{1}{x^2}\,dx = \frac{1}{2}\int_1^3 x^{-2}\,dx = \frac{1}{2}\left(-x^{-1}\right)\Big|_1^3$$

$$= \frac{1}{2}\left(-\frac{1}{x}\right)\Big|_1^3 = \left(\frac{1}{2}\cdot -\frac{1}{3}\right) - \left(\frac{1}{2}\cdot -\frac{1}{1}\right) = -\frac{1}{6} + \frac{1}{2}$$

$$= \boxed{\frac{1}{3}}$$

2. At time $t = 0$ hours, Sue begins to study for her final. After t hours, she reads at a rate of $30 - (t-2)^2$ pages per hour. Suppose that Sue studies for 5 hours.

(a) [**4 points**] In total, how many pages does Sue read?

$$\int_0^5 30 - (t-2)^2\,dt = \int 30 - w^2\,dw$$

$$= \cancel{30t - \frac{(t-2)^3}{3}} \qquad = 30w - \frac{w^3}{3}$$

$w = t-2$

$\frac{dw}{dt} = 1$

$dw = dt$

$$= 30(t-2) - \frac{(t-2)^3}{3}\Big|_0^5$$

$$= \left(30\cdot(5-2) - \frac{(5-2)^3}{3}\right) - \left(30(0-2) - \frac{(0-2)^3}{3}\right)$$

$$= \left(30\cdot 3 - \frac{3^3}{3}\right) - \left(30\cdot(-2) - \frac{(-2)^3}{3}\right)$$

$$= (90 - 9) - \left(-60 + \frac{8}{3}\right) = 81 + 60 - \frac{8}{3} = \boxed{138.\overline{3}\ \text{pages}}$$

(b) [**1 point**] On average, how many pages per hour does Sue read?

$$\frac{138.\overline{3}\ \text{pages}}{5\ \text{hours}} = \boxed{27.\overline{6}\ \text{pages per hour}}$$