Math122 **Quiz #5** **3 November 2011**

Name: Solutions

**Directions:** Show all work. No credit for answers without work.

1. [**2 points**] Find all inflection points of $f(x) = x^5 - 2x^4 + 18x$.

$f'(x) = 5x^4 - 8x^3 + 18$

$f''(x) = 20x^3 - 24x^2$

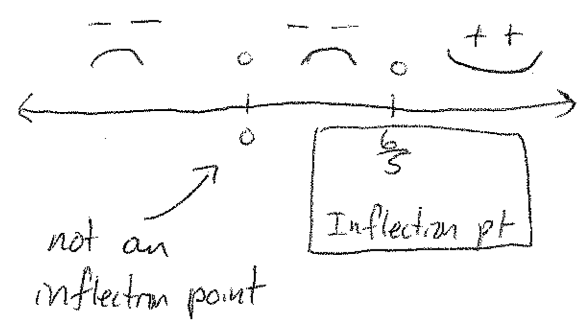
$= 4x^2(5x-6)$

Inflection pt candidates:

$x^2 = 0$ or $5x - 6 = 0$

$x = 0$ $\qquad x = \frac{6}{5}$

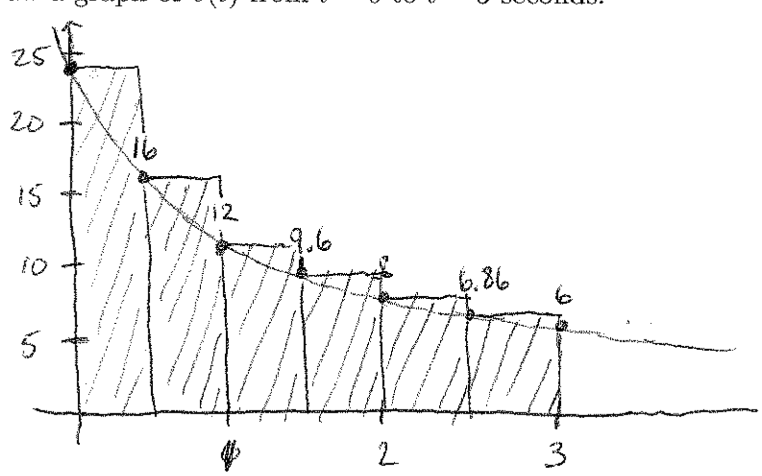
$f''(x)$ sign chart:




2. [**3 points**] Your velocity $v$ (in m/sec) at time $t$ (in seconds) is given by $v(t) = \dfrac{24}{1+t}$.

Skipped this for Quiz5. Perhaps on your test or final...

(a) Carefully draw a graph of $v(t)$ from $t = 0$ to $t = 3$ seconds.




(b) Using 6 rectangles, estimate the distance traveled between time $t = 0$ and $t = 3$.

LHS: Distance $\approx 25\cdot\frac{1}{2} + 16\cdot\frac{1}{2} + 12\cdot\frac{1}{2} + 9.6\cdot\frac{1}{2} + 8\cdot\frac{1}{2} + 6.86\cdot\frac{1}{2}$

$= \boxed{38.73 \text{ meters}}$

RHS: Distance $= 16\cdot\frac{1}{2} + 12\cdot\frac{1}{2} + 9.6\cdot\frac{1}{2} + 8\cdot\frac{1}{2} + 6.86\cdot\frac{1}{2} + 6\cdot\frac{1}{2}$

$= \boxed{29.23 \text{ meters}}$

Either answer OK

(c) Illustrate your estimate on the graph drawn in part (a). Is your estimate lower, higher, or exactly equal to the true distance traveled?

- $\boxed{29.23 \le \text{True distance} \le 38.73}$
- In the graph, we illustrate the rectangles from LHS ($\approx 38.73$m)

3. [~~2~~ **points**] Find the exact global maximum and minimum values of $g(x) = \dfrac{x}{1+x^2}$ over $[0, 10]$.

$$g'(x) = \frac{(1+x^2)\cdot 1 - x \cdot 2x}{(1+x^2)^2}$$

$$= \frac{1+x^2-2x^2}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2}$$

$$\frac{1-x^2}{(1+x^2)^2} = 0$$

$$1 - x^2 = 0$$

$$(x+1)(x-1) = 0$$

• $x = -1$ or $x = 1$

• Check: $g(0), g(1), g(10)$.

• $g(0) = \frac{0}{1+0^2} = \boxed{0 \leftarrow \text{global min}}$

• $g(1) = \frac{1}{1+1^2} = \boxed{\frac{1}{2} \leftarrow \text{global max}}$

• $g(10) = \frac{10}{1+10^2} = 0.099$

4. [**3 points**] An ice cream company finds that at a price of \$3.50, demand is 5000 units. For every \$0.15 increase in price, demand decreases by 153 units.

(a) Find the demand $q$ as a function of price $p$.

Point: $(p_0, q_0) = (3.5, 5000)$

Slope: $m = \frac{\Delta q}{\Delta p} = \frac{-153}{0.15} \approx -1020$

$$q - q_0 = m(p - p_0)$$

$$q - 5000 = -1020(p - 3.5)$$

$$q - 5000 = -1020p + 3570$$

$$\boxed{q = -1020p + 8570}$$

(b) Find the revenue $R$ as a function of price $p$.

$$R = p \cdot q$$

$$R = p(-1020p + 8570) = -1020p^2 + 8570p$$

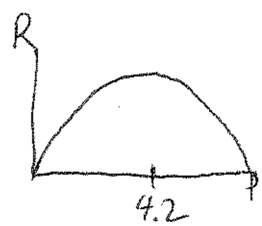
(c) Find the price and quantity sold that maximize revenue.

Find global max of $R(p) = -1020p^2 + 8570p$ over $(0, \infty)$:

$$R'(p) = -2040p + 8570$$

$$-2040p + 8570 = 0$$

$$p = \frac{-8570}{-2040} = 4.20$$




global max.

$$q = -1020(4.2) + 8570$$
$$= 4286.$$

So the price and quantity that maximize revenue are $\boxed{\$4.20}$ and $\boxed{4{,}286}$ units, respectively