Name: Solution

Directions: Show all work. No credit for answers without work.

1. **[6 parts; 0.5 points each]** Find the derivative of the given functions. No need to show work.

(a) $y = x^5$

$\frac{d}{dx}\left[x^5\right] = 5x^4$

(b) $f(x) = x^2 + \frac{1}{x^3}$ $= x^2 + x^{-3}$

$\boxed{2x - 3x^{-4}}$

(c) $y = \sqrt{t}(t+1)$ $= \sqrt{t}\cdot t + \sqrt{t}$

$= t^{3/2} + t^{1/2}$

$\boxed{\frac{3}{2}t^{1/2} + \frac{1}{2}t^{-1/2}}$

(d) $f(r) = 3r^{\sqrt{2}}$

$\boxed{3\sqrt{2}\, r^{\sqrt{2}-1}}$

(e) $y = 5^t + 2e^{3t} + e^2$

$\boxed{\ln(5)\cdot 5^t + 6e^{3t}}$

(f) $g(s) = \ln(s) - e^s$

$\boxed{\frac{1}{s} - e^s}$

2. **[1 point]** Find the equation of the line tangent to the curve $f(x) = x^2 + \ln(x)$ at $x = 3$.

① Find point (x_0, y_0).

$x_0 = 3$

$y_0 = f(3) = 3^2 + \ln(3) = 9 + \ln(3)$

② Find slope.

$f'(x) = \frac{d}{dx}\left[x^2 + \ln x\right]$

$= \frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[\ln x\right]$

$= 2x + \frac{1}{x}$

$m = f'(3) = 2\cdot 3 + \frac{1}{3} = 6 + \frac{1}{3} = \frac{19}{3}$

③ Plug into point-slope:

$y - y_0 = m(x - x_0)$

$y - (9 + \ln(3)) = \frac{19}{3}(x - 3)$

$y = \frac{19}{3}x - 19 + (9 + \ln(3))$

$\boxed{y = \frac{19}{3}x - 10 + \ln(3)}$

3. **[4 parts; 1.5 points each]** Find the derivative of the given functions.

(a) $f(t) = (t^3+t)^{61}$

$$f'(t) = \frac{d}{dt}\left[(t^3+t)^{61}\right]$$

$$= 61(t^3+t)^{60} \cdot \frac{d}{dt}\left[t^3+t\right]$$

$$= \boxed{61(t^3+t)^{60} \cdot (3t^2+1)}$$

(b) $f(x) = \ln\left(1+e^{x^2}\right)$

$$f'(x) = \frac{d}{dx}\left[\ln(1+e^{x^2})\right]$$

chain rule

$$= \frac{1}{1+e^{x^2}} \cdot \frac{d}{dx}\left[1+e^{x^2}\right]$$

$$= \frac{1}{1+e^{x^2}} \cdot \left(0 + \frac{d}{dx}\left[e^{x^2}\right]\right)$$

chain rule again!

$$= \frac{1}{1+e^{x^2}} \cdot \left(e^{x^2} \cdot \frac{d}{dx}\left[x^2\right]\right)$$

$$= \frac{1}{1+e^{x^2}} \cdot \left(e^{x^2} \cdot 2x\right)$$

$$= \boxed{\frac{2xe^{x^2}}{1+e^{x^2}}}$$

(c) $f(p) = \cancel{7p+}\ 2^p \ln(3p+4)$

$$f'(p) = \frac{d}{dp}\left[2^p \cdot \ln(3p+4)\right]$$

$$= \frac{d}{dp}\left[2^p\right] \cdot \ln(3p+4) + 2^p \cdot \frac{d}{dp}\left[\ln(3p+4)\right]$$

$$= (\ln 2) \cdot 2^p \cdot \ln(3p+4) + 2^p \cdot \frac{1}{3p+4} \cdot \frac{d}{dp}\left[3p\right.$$

$$= (\ln(2)) \cdot 2^p \cdot \ln(3p+4) + 2^p \cdot \frac{1}{3p+4} \cdot 3$$

$$= \boxed{(\ln 2) \cdot 2^p \ln(3p+4) + \frac{3 \cdot 2^p}{3p+4}}$$

(d) $f(x) = \dfrac{x^2+1}{2x - \cancel{\ln(x)}\ 1}$

$$f'(x) = \frac{(2x - \cancel{\ln(x)}\ 1) \cdot \frac{d}{dx}\left[x^2+1\right] - (x^2+1)\frac{d}{dx}\left[2x - \cancel{\ln}\ 1\right]}{(2x - \cancel{\ln(x)}\ 1)^2}$$

$$= \frac{(2x - \cancel{\ln(x)}\ 1) \cdot 2x - (x^2+1) \cdot (2)}{(2x-1)^2}$$

$$= \frac{4x^2 - 2x - 2x^2 - 2}{(2x-1)^2}$$

$$= \boxed{\frac{2(x^2-x-1)}{(2x-1)^2}}$$