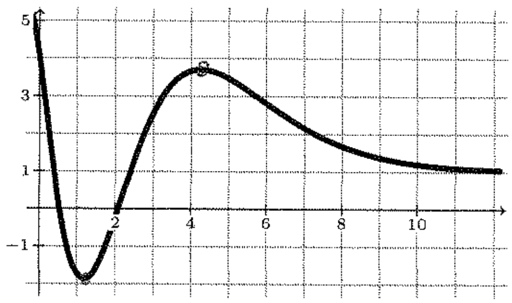
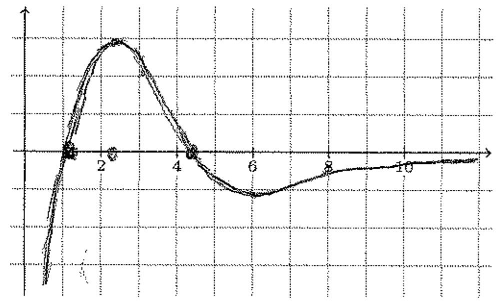
Name: Solution Key

Directions: Show all work. No credit for answers without work.

1. The graph of $f(x)$ appears below.





(a) [**1 point**] Estimate sthe point(s) x such that $f'(x) = 0$.

$\boxed{x \approx 1.2 \text{ and } x \approx 4.3}$

(b) [**2 points**] Sketch the derivative $f'(x)$ in the space provided. Your sketch should capture the important features of $f'(x)$, including the ranges over which $f'(x)$ is positive, negative, increasing, and decreasing.

2. [**4 parts, 1 point each**] The quantity q (in thousands) of radios sold depends on the price p (in dollars). Let $q = f(p)$.

(a) Translate to English: $f(60) = 80$. Be sure to include units.

When radios cost \$60, 80,000 radios are sold.

(b) Translate to English: $f'(60) = -4$. Be sure to include units.

When radios cost \$60, the amount sold decreases as the ~~other~~ price increases at a rate of 4 thousand ~~per dollar~~ radios per dollar.

(c) Estimate the number of radios sold if the price is \$61.

$f(61) \approx f(60) + 1 \cdot (-4) = 80 - 4 = \boxed{76 \text{ thousand radios}}$

(d) Estimate the number of radios sold if the price is \$58.

$$f(58) \approx f(60) + \Delta p \cdot f'(60)$$
$$\approx 80 + (-2) \cdot (-4) \approx \boxed{88 \text{ thousand radios}}$$

3. **[3 parts, 1 point each]** Let $f(x) = 2x^2$.

(a) Find the average rate of change in f over $[3, 4]$.

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$$ARC = \frac{f(4) - f(3)}{4 - 3} = \frac{2(4^2) - 2(3^2)}{1}$$

$$= 2 \cdot 16 - 2 \cdot 9$$

$$= 2(16 - 9) =$$

$$= 2 \cdot 7 = \boxed{14}$$

(b) Find the average rate of change in f over $[3, 3 + h]$.

$$ARC = \frac{f(3+h) - f(3)}{3+h-3} = \frac{2(3+h)^2 - 2 \cdot (3^2)}{h}$$

$$= \frac{2(9 + 6h + h^2) - 2 \cdot 9}{h}$$

$$= \frac{\cancel{18} + 12h + 2h^2 - \cancel{18}}{h} = \frac{12h + 2h^2}{h} = \boxed{12 + 2h}$$

(c) Use part (b) to find $f'(3)$.

$f'(3)$ = IRC of f at $x = 3$

= limit of ARC of f over $[3, 3+h]$ as $h \to 0$.

$= 12 + 2 \cdot h$ as $h \to 0$

$= \boxed{12}$.